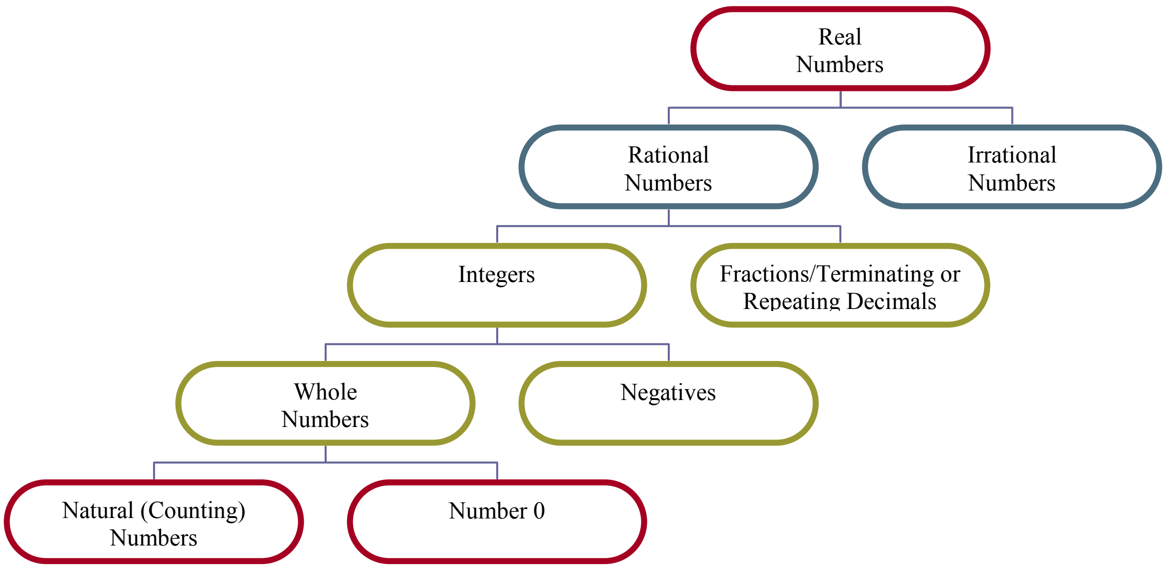
Numbers Sets

Natural Numbers: 1, 2, 3…
Whole Numbers: 0, 1, 2, 3… (Naturals & 0)
Integers: … -3, -2, -1, 0, 1, 2, 3… (Whole #s & "opposites")
Rational Numbers: $\frac{a}{b}$ where a is an Integer and b is a Natural Number (Integers, Fractions, and terminating or repeating decimals)
Irrational Numbers: decimals that do not repeat or terminate, such as $\sqrt{2}$, π , e
Real Numbers: Rational & Irrational Numbers



Operations

Addition
If the 2 numbers have the same sign, add the numbers & carry over the sign.
If the 2 numbers have different signs, subtract the smaller absolute value from the larger absolute value & carry over the sign from the larger number.

Subtraction (or adding a negative)
If subtracting a positive number, this is the same as adding a negative number.
If subtracting a negative number, this is the same as adding a positive number.

Multiplication & Division
If the 2 numbers have the same sign, the result is positive.
If the 2 numbers have different signs, the result is negative.

Division: If a & b are real numbers, and b ≠ 0, then $a \div b = a \times \frac{1}{b}$, where $\frac{1}{b}$ is called the reciprocal of b.

Absolute Value

The **absolute value** of a number is its distance from zero on the number line, denoted by | |. |1| = 1, |-3| = 3, |-x| = |x|

If k is a Real Number, then |k| = |-k| = k, if k ≥ 0 & |k| = 0, if k = 0

The **distance** between 2 numbers on the number line is the absolute value of their difference.

The distance between 3 & -4 is |3 – (-4)| = |3 + 4| = |7| = 7, or |-4 – 3| = |-7| = 7.

Exponents

Exponents are shorthand notation for repeated multiplication. The factor that is repeated is called the base; the number of times it is repeated is called the exponent. In the example below, the base is 3 and the exponent is 4, and we say "3 raised to the 4th power": $3 * 3 * 3 * 3 = 3^4$

When a number is raised to the 1st power, we usually do not write the 1, $2^1 = 2$
When a number is raised to the 2nd power, we usually say that number is squared.
When a number is raised to the 3rd power, we usually say that number is cubed.
When a negative number is raised to an even power, the result is positive.
When a negative number is raised to an odd power, the result is negative.

WARNING! $-2^2 = -4$ & $(-2)^2 = +4$. This is because of the Order of Operations (see below), Exponents are evaluated before Subtraction is performed, in this case the application of the minus sign!

Roots or Radicals

The symbol $\sqrt{\ }$ is used to denote the square root of what is inside. $\sqrt{4} = 2$, means that when 2 is squared we get 4. When no sign, + or -, is in front of the radical we want the positive root.

The symbol is also called a radical sign, $\sqrt[a]{b}$, this time we want to know what number raised to the a power will give us b, or we can say what is the a^th root of b, for some whole number a, and some real number b.

When looking for a square or other even root, we cannot have a negative under the radical and have a real number solution.

Order of Operations

Please **E**xcuse **M**y **D**ear **A**unt **S**ally

First do what is "inside **P**arentheses", or other grouping symbols, ( ), [ ], { }, | |, /

IF | |, absolute value, evaluate the | | next.

If / work top & bottom separately & do the division last.

Then work with the **E**xponential parts or Roots

Multiplication and **D**ivision are worked next from LEFT to RIGHT!

Lastly **A**ddition & **S**ubtraction is performed, again from LEFT to RIGHT!

Example:

$$3*5 - 7 + 6 \div 2 * 3 \xrightarrow{\text{1st is Multiplication 3*5}} 15 - 7 + 6 \div 2 * 3 \xrightarrow{\text{Now division } 6\div 2} 15 - 7 + 3 * 3$$

$$\xrightarrow{\text{Next is Multiplication 3*3}} 15 - 7 + 9 \xrightarrow{\text{Now Subtract 15 - 7}} 8 + 9 \xrightarrow{\text{Finally we add 8+9}} 17$$

Operations on Fractions

Let a, b, c, d be nonzero integers, if zero see appropriate rules above.

Addition(Subtraction): $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$, $\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$ and then you MUST reduce, alternatively you can find the LCM for b & d before adding, which is the Least Common Denominator(LCD) for the 2 fractions.

Multiplication: $\frac{a}{b} * \frac{c}{d} = \frac{ac}{bd}$

Division(invert & multiply): $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} * \frac{d}{c} = \frac{ad}{bc}$

Equivalent Fractions: $\frac{a}{b} = \frac{-a}{-b}$, also $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

For additional information and examples see Review of Real Number Fractions.

Properties of the Real Numbers

Addition

Commutative Property: a + b = b + a
Associative Property: (a + b) + c = a + (b + c)
Additive Identity: a + 0 = 0 + a = a
Additive Inverse (Opposites): a + (-a) = (-a) + a = 0

Multiplication

Commutative Property: a * b = b * a
Associative Property: (a * b) * c = a * (b * c)
Property of Zero: a * 0 = 0 * a = 0
Multiplicative Identity: a * 1 = 1 * a = a
Multiplicative Inverse (Reciprocals): $a * \frac{1}{a} = \frac{1}{a} * a = 1$

Division

Properties of 1
- The quotient of any number and that same number is 1: 6/6 = 1.
- The quotient of any number and 1 is that same number: 6/1 = 6.

Properties of 0
- The quotient of 0 and any number (except 0) is 0: 0/6 = 0
- The quotient of any number and 0 is not a number, 6/0 is undefined.

Distributive Property (of multiplication over addition)

a(b + c) = ab + ac = ba + ca = (b + c)a

Example: 3*83 → 3(80 + 3) → 3*80 + 3*3 → 240 + 9 → 249