Applications of Linear Equations in 2 Variables

Follow the basic steps for solving problems.

1) read & understand the problem
    a. drawing diagrams helps to organize the information for clarification
2) List the information you are given
    a. Charts sometimes makes this easier
    b. If the is a value you need to find label it as a variable
    c. Is there a basic formula you can use? You may need to create an equation from the information given, translating sentences into equations.
3) Use the information from 1 & 2 to work through the problem
    a. Be patient, check your work.
4) Interpret your results
    a. Does the solution make sense in the problem?
    b. Does the solution work in all of the equations?
    c. Remember to include the units in your answer if necessary.

Example 1:

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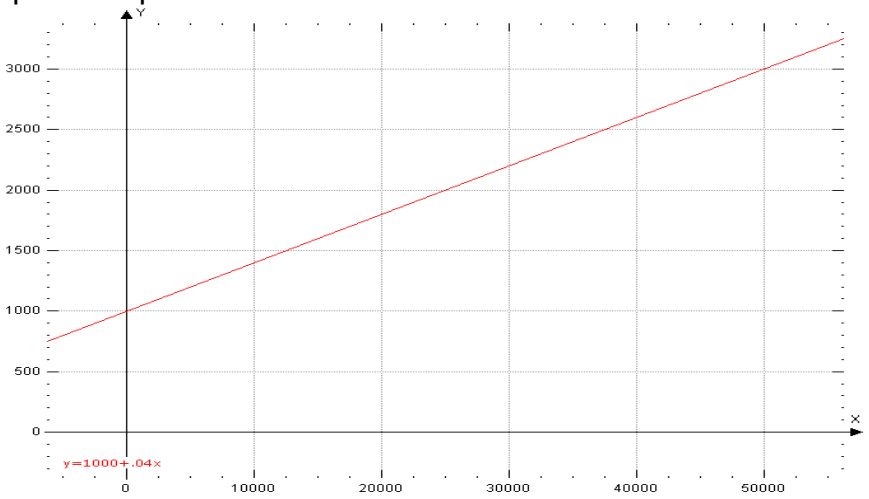
Alex is a sales rep & earns a base salary of $1000 per month plus a 4% commission on his sales for the month.

a) Write a linear equation that represents Alex's monthly salary, y, in terms of his sales, x.

His monthly salary is his base salary plus his commission. His commission is his commission rate times his sales. So his monthly salary is his base plus his commission rate times his sales. So we have y = 1000 + .04x. In slope-intercept form we have y = .04x + 1000.

b) Graph the equation:



c) What is the y-intercept and what does it represent in the context of this problem?

The y-intercept is where the graph crosses the y axis. In this case, it is (0, 1000). If Alex makes no sales for a month, he will still receive his base salary of $1000.

d) What is the slope of the line and what does it represent in the context of this problem?

The slope of the line is .04. in this case, for every $1 in sales, Alex earns 4 cents, or we can say for every $100 in sales, Alex earns $4.

e) How much will Alex make if his sales for a given month are $30,000?

a. y = .04x + 1000 → y = .04(30000) + 1000 → y = 1200 + 1000 → y = 2200
Alex will earn $220o if his sales are $30,000.

Example 2:

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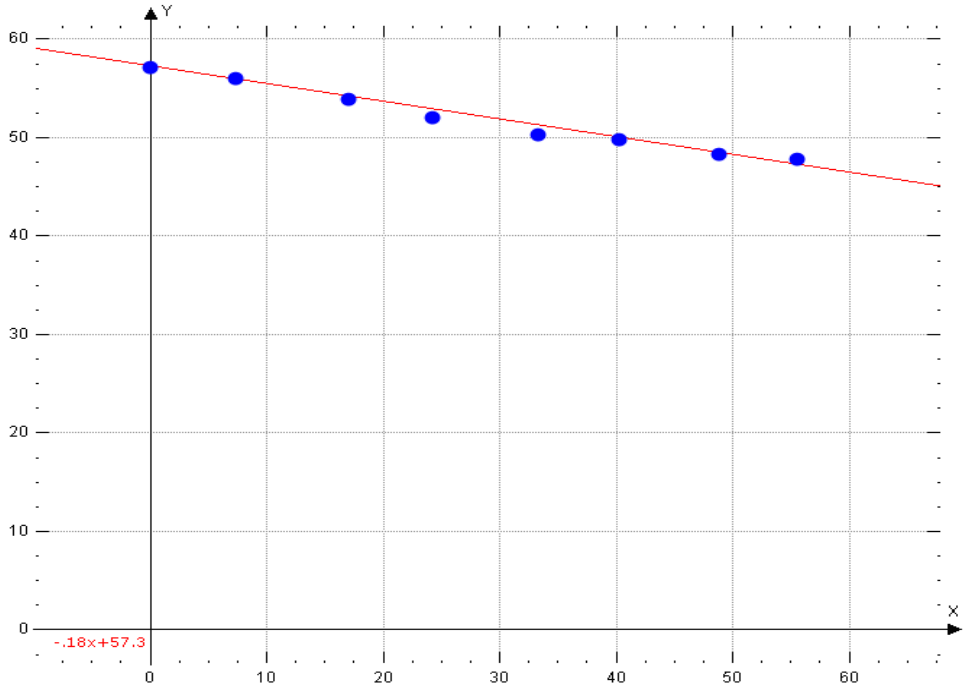
The figure the winning time for the men's 100-m freestyle swimming even for selected Olympic games.



a) Let y represent the winning time. Let x represent the number of years since 1948. Use the ordered pairs (0, 57.3) & (48, 48.7) to find a linear equation to estimate the winning time for the men's 100-m freestyle vs. the year.
   a. We need to find the slope given the 2 points above.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{48.7 - 57.3}{48 - 0} \rightarrow m = \frac{-8.6}{48} \rightarrow m \approx -.18$$

   b. With the slope we can use the y-intercept, (o, 57.3) to write the equation: y=-.18x + 57.3

b) Use this linear equation to approximate the winning time for the year 1972 & compare it with the actual time of 51.2 sec.
   a. First we need to convert 1972 to x, the number of years after 1948, so 1972 – 1948 = 24.
   b. Use x = 24 in y = -.18x + 57.3 → y = -.18(24) + 57.3 → y = 52.98 sec. In some situations this is considered pretty close to the actual time of 51.2 sec.

c) Use the linear equation to approximate the winning time for the year 1988.
   a. 1988 – 1948 = 40
   b. y = -.18x + 57.3 → y = -.18(40) + 57.3 → y = 50.1 sec.

d) What is the slope and what does it mean in this problem.
    a. The slope is -.18
    b. This means that there is a decrease (neg. sign) of .18 seconds each year.
e) Interpret the meaning of the x-intercept. Explain why the times will never "reach" the x-intercept. Do you think this trend will continue for the next 50 years or will it level off in the future? Explain.
    a. The x-intercept in this problem would mean that in some future year it would take 0 seconds for the 100-m freestyle. Which is impossible since the person would have to finish at the exact instant the he started. (318⅓,0)
    b. I don't think it will continue. Keep in mind even a car or a jet takes time to go 100 meters. So at some point, men(men's freestyle) will go as fast as humanly possible, which means they cannot reach 0 as stated in part a. You may explain it differently, there are very few wrong answers when interpreting what you think…

Another application for Slope and Linear Equations is determining if points are collinear.

Points are **collinear** if they all lie on the same line. There are a couple of ways to determine if points are collinear, usually you work with more than two points, since any two points determine a line.

One way to determine if a third point is on the line is to write the equation of the line for the first two points. Then plug the third point into the equation. If you get a true statement it is on the line. If you get a false statement, it is not on the line.

Example:
Determine if the points P_1 = (0, -5); P_2 = (9, 22); P_3 = (3, 4)
Using the first two points I can find write the equation of the line:

The slope of the line is: $m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{22 - ^- 5}{9 - 0} \rightarrow m = \frac{27}{9}$ → Reduce → m = 3.

Using point 1, the y –intercept, we get the equation: y = 3x – 5

Now, if we put the x-value of P_3 into this equation we have: y = 3(3) – 5 → y = 4. This is the y-value of P_3, so the three points are collinear. There are a couple of other things we could have done instead, but I feel this was the simplest method for this type of problem.

Another way to determine if three points are collinear is to see if each two points have the same slope. This means you will be calculating the slope three times!

Example:
Determine if the points P_1 = (-4, 5); P_2 = (8, -31); P_3 = (-2, -1)

Slope for P_1 & P_2: $m = \frac{-31 - 5}{8 - ^- 4} \rightarrow m = \frac{-36}{12}$, so m = -3

Slope for P_1 & P_3: $m = \frac{-1 - 5}{-2 - ^- 4} \rightarrow m = \frac{-6}{2}$, so m = -3

Slope for P_2 & P_3: $m = \frac{-1 - ^- 31}{-2 - 8} \rightarrow m = \frac{30}{-10}$, so m = -3

Since all three pairs of points have the same slope, they are all on the same line.

WARNING: If you forget one of the pairs, you might have parallel lines!