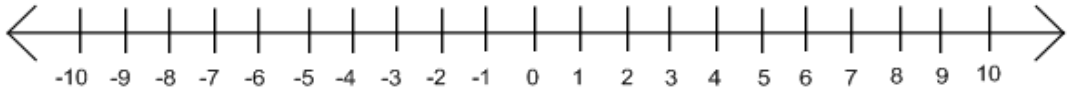
Inequalities

Symbols

< less than
> greater than
≤ less than or equal to
≥ greater than or equal to



The numbers to the left are less than the numbers to the right, -9 < 6

The numbers to the right are greater than the numbers to the left, 4 > 2

If something is at least a number, then it is greater than or equal to it, i.e. “it is at least 72°” then we could write: temperature ≥ 72° or 72° ≤ temperature

Properties of Inequality

Addition (Subtraction): if a > b, then a + c > b + c

Multiplication (Division): if a > b and c > 0, then ac > bc

if a > b and c < 0, then ac < bc (if b < a and c < 0, then bc > ac)

An example for why you must change direction when multiplying by a negative number: 3 > 2 multiply both sides by (-1) & you have -3 > -2, not true, so you must change it to -3 < -2.

Steps for Solving Inequalities

1) If it is a compound inequality (covered more in Chapter 9) break it into 2 inequalities.
2) If the inequality involves a fraction, multiply both sides by the LCD.
3) Use the Distributive property to get rid of any parenthesis.
4) Simplify each side, combine like terms.
5) Get all the variable terms to one side and all non-variable(constants) to the other side & simplify
6) Get variables alone using the multiplication property.
7) Check solution by substituting back into original, does it makes sense?

Graphing

1.) Use '(' or ')' on the endpoint(s) not included in the solution, strict inequality, see the Interval Notation File or below.

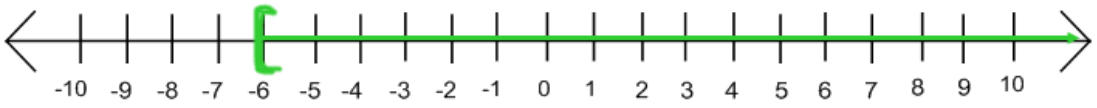
2.) Use '[' or ']' on the endpoint(s) included in the solution,

3.) Pick a point on each side of the endpoint(s) to check which way the line goes.

Examples:

1) $x \geq -6$

$-6 \leq x$



2) x – 4 > 2 – x

With this problem, we start at step 5, move the x to the left and the 4 to the right.

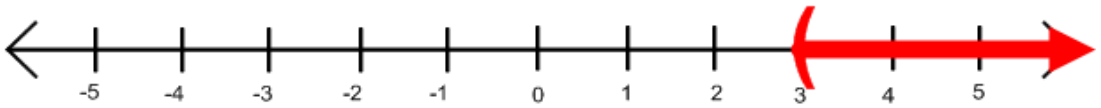
x – 4 > 2 – x
+ x + 4 + 4 + x
2x > 6

Now we just divide both sides by 2.

x > 3

Interval Notation: $(3, \infty)$

Graph:



3) 3(t + 4) ≤ t + 10

First we need to use the distributive property.

3t + 12 ≤ t + 10
- t – 12 - t – 12
2t ≤ - 2

Next subtract t and 12 from each side.

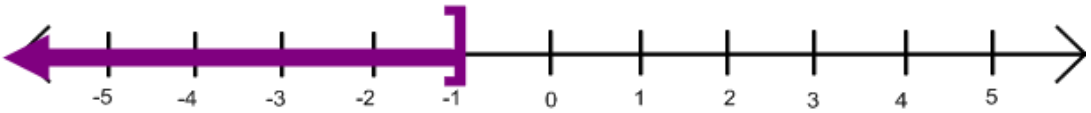
$\frac{2t}{2} \leq \frac{-2}{2}$

Divide each side by 2, to find t.

t ≤ -1

Interval Notation: $(-\infty, -1]$

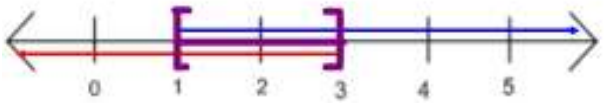
Graph:



4) $-6 \le 3k - 9 \le 0$ I recommend making this problem 2 inequalities, $-6 \le 3k - 9$ and $3k - 9 \le 0$. This cuts down on the possibility for the problems mentioned above. Many errors occur when students forget that each part of the inequality must use the addition or multiplication property.

$$\begin{array}{rrr} -6 \le 3k & -9 \le 0 \\ +9 \quad & +9 & +9 \\ \hline +3 \le 3k & & \le 9 \\ +3 \le \underline{3k} & & \le \underline{9} \\ 3 \quad 3 & & 3 \\ 1 \le k & & \le 3 \end{array}$$

$$\begin{array}{lcl} -6 \le 3k - 9 & \& & 3k - 9 \le 0 \\ 3 \le 3k & \& & 3k \le 9 \\ 1 \le k & \& & k \le 3 \end{array}$$



5) $-1 < -2x + 4 < 5$

$-4 \qquad -4 \qquad -4$

$-5 < -2x < 1$

$\frac{-5}{-2} > x > \frac{1}{-2}$ Now divide by -2 AND turn the symbols!

$\frac{5}{2} > x > -\frac{1}{2}$ or $-\frac{1}{2} < x < \frac{5}{2}$

This type of problem can be done "all at once", you just need to pay attention to every step you take!

Graph:

6) To get a "B" in a course, a student needs an average of at least an 80% but less than a 90% on 6 tests. Rowena received an 82, 76, 83, 92 and 67 (each out of 100) on the first five tests. What does she need on the sixth test to get a "B" in the course?

What we know:

This is an average problem, the average formula for this problem: $\frac{\text{sum of numbers}}{6}$

Let x be the unknown test grade.
She needs at least an 80 → 80 ≤ average.
She needs less than a 90 → average ≤ 90.

The "average" can be replaced by the formula in line 1, and the last two combined:

$$80 \le \frac{82+76+83+92+67+x}{6} \le 90$$

$$80 \le \frac{82+76+83+92+67+x}{6} \le 90$$

The first step for solving this is to simplify the middle part.

$$80 \le \frac{400+x}{6} \le 90$$

Now we can multiply each part by 6, to remove the fraction.
Multiply.

$6*80 \le 400 + x \le 6*90$

$480 \le 400 + x \le 540$

Finally subtract 400 from each side.

$-400 \quad -400 \quad -400$

$80 \le x \le 140$

Since the test is out of 100 she will not get more than 100.

$80 \le x \le 100$

Interval Notation: [80, 100]

Notations

Inequality Notation	Interval Notation	Meaning in Words
$x < b$	$(-\infty, b)$	All numbers less than b, but not including b.
$x \leq b$	$(-\infty, b]$	All numbers less than b, including b.
$a < x$	$(a, +\infty)$	All numbers greater than a, but not including a.
$a \leq x$	$[a, +\infty)$	All numbers greater than a, including a.
$a < x < b$	(a, b)	All numbers between a & b, but not including a or b.
$a \leq x < b$	$[a, b)$	All numbers between a & b, including a.
$a < x \leq b$	$(a, b]$	All numbers between a & b, including b.
$a \leq x \leq b$	$[a, b]$	All numbers between a & b, including a & b.
$x < a$ & $b < x$	$(-\infty, a) \cup (b, +\infty)$	All numbers less than a AND All numbers greater than b, but not including a & b.
$x \leq a$ & $b \leq x$	$(-\infty, a] \cup [b, +\infty)$	All numbers less than a AND All numbers greater than b, including a & b.

Be very careful on the last 2. You may come across something like: $x < 3$ & $3 < x$. This does NOT mean $x < 3 < x$! Many people get this wrong. In this case we have $x \neq 3$. You may also come across $x < 5$, $10 < x$, again do not write $10 < x < 5$, because that would mean $10 < 5$, which we all know is not true.