Audio will start at class time.

If you are not familiar with a standard deck of cards, check out: http://en.wikipedia.org/wiki/Standard_52-card_deck
We will be referring to them tonight.

More information & the PDF of this PowerPoint is available at:
http://www.ramshillfarm.com/Math/Math150/Unit_7.html

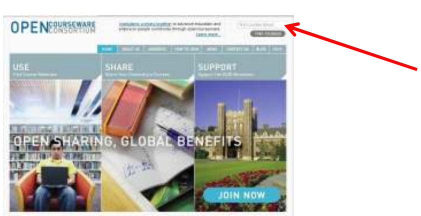
Virtual Field Trip to: http://www.oercommons.org/

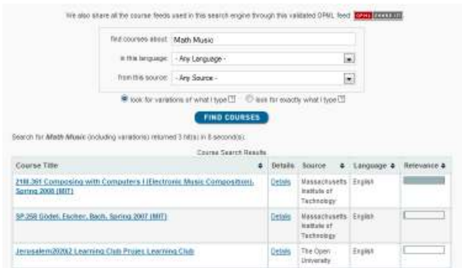
This is a search engine for Open Educational Resources (free to learn from materials). It will search many different locations.

When you get back tell how many articles you found on your search, or bookmark it to check out later.

I found 16 in the Math & Stats area on Music, keep in mind not all of them are relevant, but it sure is a good start.

http://www.ocwconsortium.org





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Sections 7.1 & 7.2

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Definitions

- An **experiment** is a controlled operation that yields a set of results.
- The possible results of an experiment are called its **outcomes**.
- An **event** is a sub-collection of the outcomes of an experiment.

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Definitions continued

- **Empirical probability** is the relative frequency of occurrence of an event and is determined by actual observations of an experiment.
- **Theoretical probability** is determined through a study of the possible *outcomes* that can occur for the given experiment.

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Empirical Probability

$$P(E) = \frac{\text{number of times event } E \text{ has occurred}}{\text{total number of times the experiment has been performed}}$$

In 100 tosses of a fair die, 19 landed showing a 3. Find the empirical probability of the die landing showing a 3.

$$P(3) = \frac{19}{100}$$

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Equally likely outcomes

- If each outcome of an experiment has the same chance of occurring as any other outcome, they are said to be **equally likely outcomes**.
- For equally likely outcomes, the probability of Event *E* may be calculated with the following formula.

$$P(E) = \frac{\text{number of outcomes favorable to } E}{\text{total number of possible outcomes}}$$

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The **law of large numbers** states that probability statements apply in practice to a large number of trials, ***not*** to a single trial. It is the relative frequency over the long run that is accurately predictable, not individual events or precise totals.

Important Facts

- The probability of an event that cannot occur is 0.
- The probability of an event that must occur is 1.
- All probabilities are a number between 0 & 1 inclusive; that is, $0 \le P(E) \le 1$.
- The sum of the probabilities of all possible outcomes of an experiment is 1.
- P(not E) = 1 – P(E)

Example

A standard deck of cards is well shuffled. Find the probability that the card is selected.

a) a 10.
b) not a 10.
c) a heart.
d) an ace, 2 or 3.
e) diamond and spade.
f) a card greater than 4 *and* less than 7.

Example continued

a) a 10

There are four 10's in a deck of 52 cards.

$$P(10) = \frac{4}{52}$$

$$P(10) = \frac{1}{13}$$

b) not a 10

$$P(\text{not a }10) = 1 - P(10)$$

$$= 1 - \frac{1}{13}$$

$$= \frac{12}{13}$$

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Example continued

c) a heart

There are 13 hearts in the deck.

$$P(heart) = \frac{13}{52}$$

$$P(heart) = \frac{1}{4}$$

d) an ace, 2 or 3

There are 4 aces, 4 twos and 4 threes, or a total of 12 cards.

$$P(A, 2\ or\ 3) = \frac{12}{52}$$

$$P(A, 2\ or\ 3) = \frac{3}{13}$$

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Example continued

e) diamond and spade

The word *and* means both events must occur. This is not possible.

$$P(diamond\ and\ spade) = \frac{0}{52}$$

$$P(\blacklozenge\ \&\ \spadesuit) = 0$$

f) a card greater than 4 and less than 7

The cards greater than 4 and less than 7 are 5's, and 6's.

$$P(4 < x < 7) = \frac{8}{52}$$

$$P(5\ or\ 6) = \frac{2}{13}$$

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7.3

Odds

Odds Against

$$\text{Odds against event} = \frac{P(\text{event fails to occur})}{P(\text{event occurs})} = \frac{P(\text{failure})}{P(\text{success})}$$

Example: Odds Against

Find the odds against rolling a 5 on one roll of a die.

$P(5)=\frac{1}{6}$ $P(\text{fails to roll a 5})=\frac{5}{6}$

odds against rolling a 5 $\frac{\frac{5}{6}}{\frac{1}{6}} \rightarrow \frac{5}{6} \div \frac{1}{6} \rightarrow \frac{5}{6} * \frac{6}{1} \rightarrow \frac{5}{1}$

The odds against rolling a 5 are 5:1.

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Odds in Favor

$$\text{Odds in favor of event} = \frac{P(\text{event occurs})}{P(\text{event fails to occur})} = \frac{P(\text{success})}{P(\text{failure})}$$

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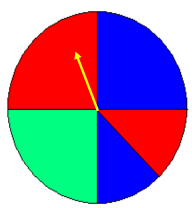
Example

Find the odds in favor of landing on **blue** in one spin of the spinner.

$$P(\text{blue}) = \frac{3}{8} \qquad P(\text{not blue}) = \frac{5}{8}$$



$$\text{odds in favor} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{8} \cdot \frac{8}{5} = \frac{3}{5}$$

The odds in favor of spinning blue are 3:5.

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Probability from Odds

The odds for spinning a blue on a certain spinner are 4:3. Find the probability that

a) blue is spun.

b) blue is not spun.

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Solution

Since the odds are 4:3 the denominators must be 4 + 3 = 7.

The probabilities ratios are:

$$P(\text{blue}) = \frac{4}{7}$$

$$P(\text{not blue}) = \frac{3}{7}$$

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7.4

Expected Value (Expectation)

Expected Value

$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + P_3 \cdot A_3 + \ldots + P_n \cdot A_n$$

P_1: probability that the first event will occur

A_1: net amount won or lost if the first event occurs.

EXAMPLE Lottery Expectations

Suppose that $1 lottery tickets have the following probabilities: 1 in 5 to win a free ticket (worth $1), 1 in 100 to win $5, 1 in 100,000 to win $1,000, and 1 in 10 million to win $1 million. What is the expected value of a lottery ticket? Discuss the implications. (Note: Winners do not get back the $1 they spend on the ticket.)

Solution: The easiest way to proceed is to make a table (next slide) of all the relevant events with their values and probabilities. We are calculating the expected value of a lottery ticket to you; thus, the ticket price has a negative value because it costs you money, while the values of the winnings are positive.

Lottery Expectations

Event	Value	Probability	Value × probability
Ticket purchase	-$1	1	$(-\$1) \times 1 = -\$1.00$
Win free ticket	$1	$\frac{1}{5}$	$\$1 \times \frac{1}{5} = \$0.20$
Win $5	$5	$\frac{1}{100}$	$\$5 \times \frac{1}{100} = \$0.05$
Win $1000	$1,000	$\frac{1}{100{,}000}$	$\$1{,}000 \times \frac{1}{100{,}000} = \$0.01$
Win $1 million	$1,000,000	$\frac{1}{10{,}000{,}000}$	$\$1{,}000{,}000 \times \frac{1}{10{,}000{,}000} = \$0.10$

The expected value is the sum of all the products *value* × *probability*, which the final column of the table shows to be –$0.64.

Thus, averaged over many tickets, you should expect to lose 64¢ for each lottery ticket that you buy. If you buy, say, 1,000 tickets, you should expect to *lose* about 1,000 × $0.64 = $640.

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7.6

Or and *And* Problems

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Or Problems

P(*A* or *B*) = *P*(*A*) + *P*(*B*) - *P*(*A* and *B*)

Each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 is written on a separate piece of paper. The 10 pieces of paper are then placed in a bowl and one is randomly selected.

a) Find the probability that the piece of paper selected contains an even number or a number greater than 5.

P(*A* or *B*) = *P*(*A*) + *P*(*B*) - *P*(*A* and *B*)

$$P\begin{pmatrix}\text{even or}\\ \text{greater than 5}\end{pmatrix} =$$

$$P(\text{even}) + P(\text{greater than 5}) - P\begin{pmatrix}\text{even and}\\ \text{greater than 5}\end{pmatrix}$$

$$= \frac{5}{10} + \frac{5}{10} - \frac{3}{10} = \frac{7}{10}$$

Thus, the probability of selecting an even number or a number greater than 5 is 7/10.

b) Find the probability that the piece of paper selected contains a number less than 3 or a number greater than 7.

$$P(\text{less than 3}) = \frac{2}{10} \qquad P(\text{greater than 7}) = \frac{3}{10}$$

There are no numbers that are *both* less than 3 and greater than 7. Therefore,

$$P\begin{pmatrix}\text{less than 3 or}\\ \text{greater than 7}\end{pmatrix} = \frac{2}{10} + \frac{3}{10} - 0 = \frac{5}{10} = \frac{1}{2}$$

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Mutually Exclusive

Two events *A* and *B* are **mutually exclusive** if it is *impossible* for both events to occur simultaneously.

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Independent Events

- Event *A* and Event *B* are **independent events** if the occurrence of either event in no way affects the probability of the occurrence of the other event.
- Experiments done with replacement will result in independent events, and those done without replacement will result in dependent events.

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And Problems

$P(A \text{ and } B) = P(A) \bullet P(B)$

Two cards are to be selected *with replacement* from a deck of cards. Find the probability that two red cards will be selected.

$$P(A) \cdot P(B) = P(\text{red}) \cdot P(\text{red})$$
$$= \frac{26}{52} \cdot \frac{26}{52}$$
$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

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Example

Two cards are to be selected *without replacement* from a deck of cards. Find the probability that two red cards will be selected.

$$P(A) \cdot P(B) = P(\text{red}) \cdot P(\text{red})$$
$$= \frac{26}{52} \cdot \frac{25}{51}$$
$$= \frac{1}{2} \cdot \frac{25}{51} = \frac{25}{102}$$

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Example

A package of 30 tulip bulbs contains 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers. Three bulbs are randomly selected and planted. Find the probability of each of the following.

a) All three bulbs produces pink flowers.
b) The first bulb selected produces a red flower, the second produces a yellow flower and the third produces a red flower.
c) None of the bulbs produces a yellow flower.
d) At least one produces yellow flowers.

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Solution

30 tulip bulbs: 14 bulbs for red flowers, 10 for yellow flowers, 6 for pink flowers

a. All three bulbs: pink flowers

$$P(3\text{ pink}) = P(\text{pink }1) \cdot P(\text{pink }2) \cdot P(\text{pink }3)$$
$$= \frac{6}{30} \cdot \frac{5}{29} \cdot \frac{4}{28}$$
$$= \frac{1}{203}$$

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Solution

30 tulip bulbs: 14 bulbs for red flowers, 10 for yellow flowers, 6 for pink flowers

b. First bulb: red; Second: yellow; Third: red

$$P(\text{red, yellow, red}) = P(\text{red}) \cdot P(\text{yellow}) \cdot P(\text{red})$$
$$= \frac{14}{30} \cdot \frac{10}{29} \cdot \frac{13}{28}$$
$$= \frac{13}{174}$$

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Solution

30 tulip bulbs: 14 bulbs for red flowers, 10 for yellow flowers, 6 for pink flowers

c. None of the bulbs produces a yellow flower.

$$P\begin{pmatrix}\text{none}\\\text{yellow}\end{pmatrix} = P\begin{pmatrix}\text{first not}\\\text{yellow}\end{pmatrix} \cdot P\begin{pmatrix}\text{second not}\\\text{yellow}\end{pmatrix} \cdot P\begin{pmatrix}\text{third not}\\\text{yellow}\end{pmatrix}$$
$$= \frac{20}{30} \cdot \frac{19}{29} \cdot \frac{18}{28}$$
$$= \frac{57}{203}$$

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Solution

30 tulip bulbs: 14 bulbs for red flowers, 10 for yellow flowers, 6 for pink flowers

d. At least one produces yellow flowers.

P(at least one yellow) = 1 - P(no yellow)

$$= 1 - \frac{57}{203}$$
$$= \frac{146}{203}$$

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Final Project Tips

General Tips

- Start Early
- If doing a paper, remember a page is a full page.
- Submit early. If it is submitted by Unit 8, I will preview it for you, if you email me too!
- Submit it to the Writing Center. They love to read papers & presentations! (In Unit 9, they are slow)
- If you put in graphics, make sure they pertain to your information, otherwise they are considered "distracting".

Content

- MS Word or PowerPoint
- Good Videos about what not to do in PowerPoint:
  - How NOT to use PowerPoint
  - Life After Death by PowerPoint
  - What Not to Do in PowerPoint
- Keep in mind this should be a short presentation to introduce others to your profession & how it uses 1 mathematical concept.
- Remember, you will be sharing this with your classmates in Unit 10, they may not know your field or how you might be able to use your topic.

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Recommended "Layout"

1. Provide your name, the project title, and the course and section number
2. Introduce your chosen profession and give a brief overview of the concept you will apply to the profession
3. Describe how the concept can apply to your chosen profession
4. Provide examples of situations in which you would use the concept you have chosen
   i. Pages 3 & 4 can be reorganized for a better "read"
5. Provide any resources you have used to give credit to others' ideas and information (keep in mind you at least read the textbook!)

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