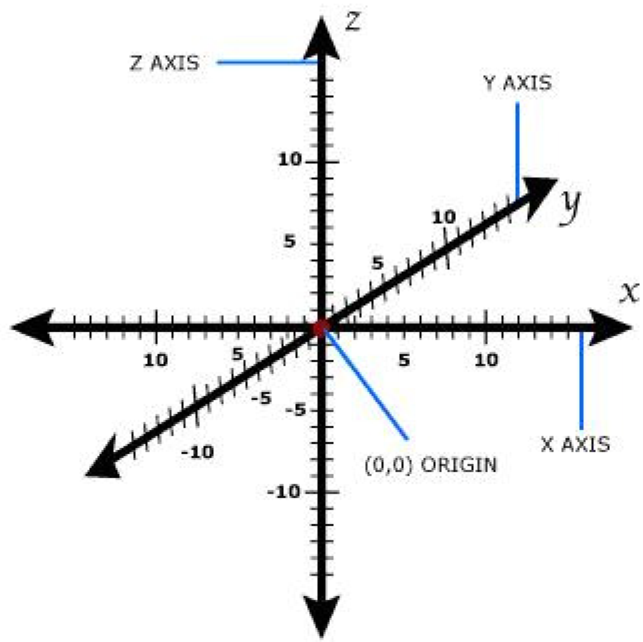
Systems of Linear Equations in Three Variables

A Linear Equation in three variables, x, y, z, in Standard Form is **Ax + By + Cz = D**, where not all A, B and C equal zero.



In order for a system of 3 equations to have a solution we must have 3 independent equations and 3 variables. The equations are independent if there is no real number, f, such that f times an equation equals one of the others.

Example:

A) $2x + 3y - z = 6$

B) $4x + 6y - 2z = 12$

Notice, if we multiply Equation A by 2, we have Equation B.

An **ordered triple** (x, y, z) is a point in the 3 dimensional or x-y-z plane.

The **solutions set** for this system is the set of all ordered triples that solve all of the equations in the system.

The following are similar to the definitions for Systems in Two Variables.

When there is exactly one solution the system is called an **independent system**:

1.) One ordered triple solution, find one value for x, one value for y, and one value for z

If there are no ordered pairs that solve both equations the system is called **inconsistent:**

1.) Zero solutions, come to a statement that is false no matter what values of x, y, or z are used.

When there is more than one solution to the system the system is called a **dependent system** of equations.

1.) Infinite solutions, come to a statement that is true no matter what values of x, y, and z are used.

Methods for Solving

Addition Method

A.) Eliminate one variable from 2 equations, using the Addition Method for Two Variables.
B.) Now treat these 2 new equations as a system of two equations in two unknowns, using the Addition Method for Two Variables. **
C.) Substitute the two values found in B into one of the original equations.
D.) Double check by putting all values into all equations.

Substitution Method

A.) Solve one of the equations for a variable

B.) Substitute it into the other 2 equations, and simplify.

C.) Now you have 2 equations, in 2 variables. Use the Substitution Method for Two Variables. **

D.) Substitute the two values found in C into one of the original equations.
E.) Double check by putting all values into all equations.

**NOTE: For Systems of Equations in more than Two Variables, you may find that it is easier to "Mix N Match" Methods. For example, once you have two equations in two unknowns, you may want to change the method that you use for solving.

WARNING: THIS IS THE ONLY PLACE YOU CAN CHANGE!

Sometimes one method is easier/shorter than the other, there is no correct choice, either one will get you the correct solution, if you don't make "silly" mistakes!

Example Using Addition Method:

1.) $x + 2y - z = 1$

2.) $2x - y + z = 6$

3.) $x + 3y - z = 2$

A) First: Eliminate one variable from 2 equations: (1. + 2. & 2. + 3.)

1.)	$x + 2y - z = 1$	2.)	$2x - y + z = 6$
2.)	$\underline{2x - y + z = 6}$	3.)	$\underline{x + 3y - z = 2}$
4.)	$3x + y \quad = 7$	5.)	$3x + 2y \quad = 8$

B) Now treat 4.) & 5.) as a system of two equations in two unknowns.

4.) $3x + y = 7$

5.) $3x + 2y = 8$

Multiply equation 5.) by -1 → $-3x - 2y = -8$

Combine 4.) & -5.) $3x + y = 7$

$$\underline{-3x - 2y = -8}$$

$$0x - y = -1$$

Solve $-y = -1$ for y, $y = 1$

Substitute $y = 1$ into equation 4.): $3x + y = 7$ → $3x + (1) = 7$

Solve for x: $3x = 6$ → $x = 2$

C) Finally, put $x = 2$ & $y = 1$ into equation 1.) & solve for z.

$(2) + 2(1) - z = 1$ → $4 - z = 1$ → $3 = z$

D) Recommend putting $x = 2$, $y = 1$, & $z = 3$ into equation 2.) or 3.) to double check.

2.) $2(2) - (1) + 3 = 6$ (true!)

Example Using the Substitution Method:

1) $x + 2y - 3z = 3$
2) $2x - y + z = -4$
3) $x + y - 4z = 1$

A) Solve equation 1) for x. We subtract (2y – 3z) from each side:

$$\begin{array}{lcl} x + 2y - 3z & = & 3 \\ \underline{\quad - 2y + 3z} & & \underline{- 2y + 3z} \\ x & = & 3 - 2y + 3z \end{array}$$

B) Substitute into equations 2 & 3, and simplify.

2) → $2(3 - 2y + 3z) - y + z = -4$ → $6 - 4y + 6z - y + z = -4$ → -6 from each side
→ $6 - 6 - 5y + 7z = -4 - 6$ → $-5y + 7z = -10$

3) → $(3 - 2y + 3z) + y - 4z = 1$ → $3 - 2y + 3z + y - 4z = 1$ → -3 from each side
→ $-y - z = -2$

C) Now we have Two Equations with only Two Variable:

4) $-5y + 7z = -10$

5) $-y - z = -2$

We can either use Substitution again or we can use the Addition Method for Two Variables, your choice. I will use Substitution Method.

Change 5) so that y is by itself → $-y = z - 2$ → $y = -z + 2$

Substitute this expression for y in Equation 4) → $-5(-z + 2) + 7z = -10$ → Simplify →

$5z - 10 + 7z = -10$ → + 10 to both sides → $12z = 0$ → $z = 0$

Now put $z = 0$ into 5) → $-y - (0) = -2$ → $y = 2$

D) I will put $z = 0$, $y = 2$ into 1) → $x + 2(2) - 3(0) = 3$ → $x + 4 = 3$ → $x = -1$.

Expected Solution: (-1, 2, 0)

E) Double Check ALL values in all equations:

1) $x + 2y - 3z = 3$ → $(-1) + 2(2) - 3(0) = 3$ TRUE
2) $2x - y + z = -4$ → $2(-1) - (2) + (0) = -4$ TRUE
3) $x + y - 4z = 1$ → $(-1) + (2) - 4(0) = 1$ TRUE

Since all 3 values make true statements in all three equations, the solution is (-1, 2, 0).