Systems of Linear Equations in Two Variables
Solution by Substitution

This method is called the Substitution Method, because you repeated substitute expressions and values into the equations until you come to a viable solution.

Substitution Method

A.) Solve one of the equations for one variable.
B.) Substitute into the other equation.
C.) Solve this second equation.
D.) Substitute this value back into the first equation
E.) Solve this equation to find the value of the other variable.
F.) Double check by putting both values into both equations.

Examples:

Ex. 1) Equation 1) $3x + 4y = 18$
Equation 2) $y = - 2x + 2$

A.) Solve one of the equations for one variable.
Equation 2 is already in this form.

B.) Substitute Equation 2 into Equation 1.
$3x + 4(- 2x + 2) = 18$, since this now has only 1 variable we can solve it as we did in Chapter 1.

C.) Solve this new equation, in this case for x.

$3x + 4(- 2x + 2) = 18$	Distribute the 4 on the left.
$3x - 8x + 8 = 18$	Combine like terms.
$- 5x + 8 = 18$	
$- 5x = 10$	Subtract 8 from each side.
$\frac{-5x}{-5} = \frac{10}{-5}$	Divide both sides by -5.
$x = - 2$	Reduce.

D.) Substitute this value back into the first equation
$3(-2) + 4y = 18$

E.) Solve this equation to find the value of the other variable.

$3(-2) + 4y = 18$	Simplify.
$-6 + 4y = 18$	
$4y = 24$	Add 6 to both sides (subtract -6)
$\frac{4y}{4} = \frac{24}{4}$	Divide both sides by 4.
$y = 6$	Reduce.

F.) Double check by putting both values into both equations.

Equation 1) 3x + 4y = 18; x = - 2; y = 6	2) y = - 2x + 2; x = - 2; y = 6
3(-2) + 4(6) → -6 + 24 → 18 So this equation is true, thus (- 2, 6) is a solution to Equation 1.	-2(-2) + 2 → 4 + 2 → 6 So this equation is true, thus (- 2, 6) is a solution to Equation 2.

Since (- 2, 6) is a solution to both equations, it is a solution to the System of Equations.

Ex. 2) Equation 1) 3x + 2y = 7
Equation 2) 6x – 4y = 5

A.) Solve Equation 1) for y

New Equation 1) y = 7/2 – (3/2)x

B.) Substitute New Equation 1) into Equation 2) for y

6x – 4(3.5 – 1.5x) = 5, since this now has only 1 variable we can solve it as we did in Chapter 1.

C.) Solve for x

6x – 4(3.5 – 1.5x) = 5	Distribute -4 on the left side of the equation.
6x – 14 + 6x = 5	
12x = 19	Combine like terms.
$\frac{12x}{12} = \frac{19}{12}$	Divide both sides by 12 to get the x by itself.
$x = \frac{19}{12}$	Now we have the value for the x variable, but we are not done yet.

D.) Substitute $x = \frac{19}{12}$ into Original Equation 1, 3x + 2y = 7 (this is to help verify no errors).

$$3\left(\frac{19}{12}\right) + 2y = 7$$

E.) Solve for y

$3\left(\frac{19}{12}\right)+2y=7$	Reduce $3\left(\frac{19}{12}\right)$ to $\frac{19}{4}$ or 19.25
$\frac{19}{4}+2y=7$	Subtract $\frac{19}{4}$ from each side.
$2y=7-\frac{19}{4}$	Simplify the left side.
$2y=\frac{9}{4}$	
$\left(\frac{1}{2}\right)(2y)=\left(\frac{1}{2}\right)\left(\frac{9}{4}\right)$	Divide both sides by 2, or multiply by $\frac{1}{2}$.
$y=\frac{9}{8}$	

F.) Double check, put both values into both original equations

Equation 1) 3x + 2y = 7; $x=\frac{19}{12};y=\frac{9}{8}$	Equation 2) 6x – 4y = 5
$3\left(\frac{19}{12}\right)+2\left(\frac{9}{8}\right)\xrightarrow{Reduce}\frac{19}{4}+\frac{9}{4}\rightarrow\frac{28}{4}\rightarrow 7$	$6\left(\frac{19}{12}\right)-4\left(\frac{9}{8}\right)\xrightarrow{Reduce}\frac{19}{2}-\frac{9}{2}\rightarrow\frac{10}{2}\rightarrow 5$
So this equation is true, thus $\left(\frac{19}{12},\frac{9}{8}\right)$ is a solution to Equation 1.	So this equation is true, thus $\left(\frac{19}{12},\frac{9}{8}\right)$ is a solution to Equation 2.

Since $\left(\frac{19}{12},\frac{9}{8}\right)$ is a solution to both equations, it is a solution to the System of Equations.