Systems of Equations in Two Variables
Solution by Addition

The Addition Method is also called the Elimination Method. You add the two equations together to eliminate a variable. Then you end up with one equation and one variable, which we can solve using methods from solving Linear Equations.

Addition Method

A.) If necessary, usually is, multiply one of the equations by a number so that one of the variables will drop out when you add them.
B.) Add the two equations to remove one of the variables.
C.) Solve the resulting equation for the remaining variable.
D.) Substitute this value back into one of the ORIGINAL equations
E.) Solve for the other variable.
F.) Double check by putting both values into both equations.

Examples:

Ex 1) Equation 1) $-2a + 3b = 6$
Equation 2) $2a - 4b = -10$

A.) If necessary, usually is, multiply one of the equations by a number .
Not necessary.

B.) Add the two equations to remove one of the variables.

$$\begin{array}{lr} \text{Equation 1)} & -2a + 3b = 6 \\ \underline{\text{(+) Equation 2)}} & \underline{2a - 4b = -10} \\ & -b = -4 \end{array}$$

C.) Solve this equation for b.
$-b = -4$ → multiply both sides by (-1) → $b = 4$.

D.) Substitute this value back into one of the ORIGINAL equations
Equation 1) $-2a + 3b = 6$; $b = 4$ → $-2a + 3(4) = 6$

E.) Solve for a.

$-2a + 3(4) = 6$	Simplify.
$-2a + 12 = 6$	Subtract 12 from both sides.
$-2a = -6$	
$\frac{-2a}{-2} = \frac{-6}{-2}$	Divide both sides by (-2).
$a = 3$	Reduce.

F.) Double check by putting both values into both equations.

Equation 1) -2a + 3b = 6; a = 3; b = 4	Equation 2) 2a – 4b = - 10; a = 3; b = 4
-2(3) + 3(4) → -6 + 12 → 6 So this equation is true, thus (3, 4) is a solution to Equation 1.	2(3) – 4(4) → 6 – 16 → 10 So this equation is true, thus (3, 4) is a solution to Equation 2.

Since (3, 4) is a solution to both equations, it is a solution to the System of Equations.

Ex. 2)

Equation 1) 3x + 2y = 7
Equation 2) 6x – 4y = 5

A.) Multiply Equation 1) by 2
2(3x + 2y) = 2(7) → 6x + 4y = 14

B.) Add the two equations:

$$\begin{array}{r} 6x + 4y = 14 \\ \underline{(+)\ 6x - 4y = \ \ 5} \\ 12x + 0y = 19 \end{array}$$

C.) Solve this equation for x

12x = 19 → $x = \frac{19}{12}$

D.) Substitute $x = \frac{19}{12}$ into Equation 1

Equation 1) 3x + 2y = 7; $x = \frac{19}{12}$ → $3\left(\frac{19}{12}\right) + 2y = 7$

E.) Solve for y: 2y = 7 – 19/4 → 2y = 9/4 → y = 9/8

$3\left(\frac{19}{12}\right)+2y=7$	Reduce $3\left(\frac{19}{12}\right)$ to $\frac{19}{4}$ or 19.25
$\frac{19}{4}+2y=7$	Subtract $\frac{19}{4}$ from each side.
$2y=7-\frac{19}{4}$	Simplify the left side.
$2y=\frac{9}{4}$	Divide both sides by 2.
$\frac{2y}{2}=\frac{\left(\frac{9}{4}\right)}{2}$	Remember fraction rules $\frac{9}{4}\div 2\rightarrow\frac{9}{4}*\frac{1}{2}$.
$y=\frac{9}{8}$	

F.) Double check, put both values into both original equations

Equation 1) 3x + 2y = 7; $x=\frac{19}{12};y=\frac{9}{8}$	Equation 2) 6x – 4y = 5
$3\left(\frac{19}{12}\right)+2\left(\frac{9}{8}\right)\xrightarrow{\text{Reduce}}\frac{19}{4}+\frac{9}{4}\rightarrow\frac{28}{4}\rightarrow 7$	$6\left(\frac{19}{12}\right)-4\left(\frac{9}{8}\right)\xrightarrow{\text{Reduce}}\frac{19}{2}-\frac{9}{2}\rightarrow\frac{10}{2}\rightarrow 5$
So this equation is true, thus $\left(\frac{19}{12},\frac{9}{8}\right)$ is a solution to Equation 1.	So this equation is true, thus $\left(\frac{19}{12},\frac{9}{8}\right)$ is a solution to Equation 2.

Since $\left(\frac{19}{12},\frac{9}{8}\right)$ is a solution to both equations, it is a solution to the System of Equations.