Systems of Linear Equations in Two Variables
Applications

Example 1: Air Speed use d = rt

Airplane, distance one way = 750 mile
With the wind, time = 3 hours
Against the wind, time = 5 hours
Find the rate of the plane in calm air and the rate of the wind.
Let p = plane speed in calm air & w = wind speed

- When travelling with the wind the rate of the wind is added to the rate of the plane, it moves faster (tailwind).
- When travelling against the wind, the rate of the wind is subtracted from the rate of the plane, the wind slows the plane down (headwind).

	d	r	t	Equations d = rt
with the wind	750	p + w	3	750 = (p + w)3
against the wind	750	p - w	5	750 = (p – w)5

Reduced Equations:

1.) $250 = p + w$
2.) $150 = p - w$

The Elimination/Addition Method is the best to use, since the w's have equal & opposite coefficients.

$$\begin{array}{rl} 1.)\ 250 = p + w & \\ \underline{+\ 2.)\ 150 = p - w} & \\ 400 = 2p & \end{array}$$

adding the 2 equations together. Next we solve for p, by dividing both sides by 2

$$\frac{400}{2} = \frac{2p}{2}$$

$$200 = p$$

To find w, we substitute p = 200 into either equation 1.) or equation 2.). I will use 1.)
$250 = (200) + w$
$50 = w$ subtracting 200 from each side.

Thus, the plane's speed in calm air is 200 mph & the wind's speed is 50 mph.

Don't forget units!

Example 2: An Integer Problem

The sum of 2 numbers is 20. The larger number is 5 less than 4 times the smaller.
(Let x = the larger number & y = the smaller number)

Equations:

1.) $x + y = 20$ (sum of the numbers is 20)
2.) $x = 4y - 5$ (5 less than 4 times the smaller)

This problem is perfect for the Substitution Method since equation 2.) is in the form x =. The first step is to substitute equation 2.) into equation 1.) for x. So equation 1.) becomes:

$(4y - 5) + y = 20$	Next we simplify the left hand side of the equation.
$5y - 5 = 20$	Then solve for y.
$\underline{\quad +5 \quad +5}$	Add 5 to both sides
$5y = 25$	
$\frac{5y}{5} = \frac{25}{5}$	Divide both sides by 5
$y = 5$	Now we put this value into an original equation 1.) or 2.)
$x = 4(5) - 5$	
$x = 20 - 5$	
$x = 15$	

So now we have our 2 integers, x = 15 & y = 5.

Example 3: Water Current use d = rt

Motorboat, distance one way = 24 miles
With the current, time = 2 hours
Against the current, time = 3 hours
Find the rate of the boat in still water and the rate of the current.
Let x = rate of the boat (in still water) & y = rate of the current

Current is like the wind, only in the water instead of the air. With the current is also called downstream; against the current is called upstream.

	D	r	t	Equations d = rt
with the current	24	$x + y$	2	$24 = (x + y)2$
against the current	24	$x - y$	3	$24 = (x - y)3$

Reduced Equations:

1.) $12 = x + y$
2.) $8 = x - y$

Like Example 1, the Elimination Method is the best choice. So we add equations 1.) & 2.)

$$\begin{array}{rl} 1.) & 12 = x + y \\ +\ \underline{2.)} & \underline{8 = x - y} \\ & 20 = 2x \\ & \dfrac{20}{2} = \dfrac{2x}{2} \\ & \color{red}{10 = x} \end{array}$$

dividing both side by 2, we get a value for x, which we can substitute into either original equation.

Substituting x = 10 into eq. 2.) $8 = (10) - y$
$-2 = -y$
$\color{red}{2 = y}$

Thus we have the boats speed in still water as 10 mph & the current has a speed of 2 mph. Again the units are important.

Example 4: Total Cost

20 60-watt light bulbs & 30 fluorescent lights cost $40
30 60-watt light bulbs & 10 fluorescent lights cost $25.
Find the cost of a 60-watt bulb & the cost of a fluorescent light.
Let b = the cost of one 60-watt bulb & f = the cost of one fluorescent light

cost of all 60-watt bulbs = # of bulbs times the cost per bulb
Total cost = cost of 60-watt bulbs + cost of fluorescent lights

Purchase 1	# of bulbs	Cost per bulb	Total cost by type
60-watt	20	b	20b
Florescent	30	f	30f
Total of Purchase 1			40

Purchase 2	# of bulbs	Cost per bulb	Total cost by type
60-watt	30	b	30b
Florescent	10	f	10f
Total of Purchase 2			25

Equations are formed by adding the total cost by type to equal the Total of the purchase:

1.) $20b + 30f = 40$
2.) $30b + 10f = 25$

We can use the Elimination Method by multiplying equation 2 by (-3) then adding it to the original form of equation 1.). This is just one option for solving, there are others…

$$\begin{array}{rrrrr} & 1.)\ 20b & + & 30f & = 40 \\ \underline{+\ (-3)^*\ 2.)} & \underline{-90b} & \underline{+} & \underline{-30f} & \underline{= -75} \\ & -70b & & & = -35 \end{array}$$

$$\frac{-70b}{-70} = \frac{-35}{-70}$$

$b = .5$ We can substitute b = .5 into eq. 1.) or 2.), I will use 1.)

$b = .5 \rightarrow$ 1.) $20(.5) + 30f = 40 \rightarrow 10 + 30f = 40 \rightarrow 30f = 30 \rightarrow f = 1$

The cost for each 60-watt bulb is 50 cents & each florescent bulb is $1.