

Multiplication of Real Numbers

When we add the same positive integer to itself many times, we can use multiplication to shorten the notation for integers. $5 + 5 + 5 + 5 = 20$, we added 5 to itself 4 times, so we have 5 times 4 = $5 * 4 = 20$.

Whole Numbers

When we add the same Whole Number to itself many times, we can use multiplication to shorten the notation. $5 + 5 + 5 + 5 = 20$, we added 5 to itself 4 times, so we have 5 times 4 = $5 * 4 = 20$.

Integers

We treat the numbers as positive to perform the multiplication then apply the following rules for the sign.

If the 2 numbers have the same sign, the result is positive.

If the 2 numbers have different signs, the result is negative.

Tips:

$$+\text{number} * +\text{number} = +\text{number}$$

$$-\text{number} * -\text{number} = +\text{number}$$

$$+\text{number} * -\text{number} = -\text{number}$$

Examples:

$$6 * 7 = 42$$

$$-6 * -7 = 42$$

$$6 * -7 = -42$$

Decimals

- 1) Multiply the decimals as though they were whole numbers.
- 2) The decimal point in the product is placed so the number of decimal places in the product is equal to the *sum* of the number of decimal places in the factors.

Shortcuts with Powers of 10

(10, 100, 1000, ...) Move the decimal point to the *right* the same number of places as there are *zeros* in the power of 10.

(.1, .01, .001, ...) Move the decimal point to the *left* the same number of places as there are *decimal places* in the power of 10.

Examples:

$$\begin{array}{r} 3.1415 \\ * \quad 5 \\ \hline 15.7075 \end{array}$$

$$\begin{array}{r} 2.36 \\ * \quad 100 \\ \hline 236.00 \end{array}$$

$$\begin{array}{r} 156 \\ * \quad .001 \\ \hline 0.156 \end{array}$$

Fractions

Let a, b, c, d be nonzero integers.

Multiply the numerators and also multiply the denominators: $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$

Example:

$$\frac{3}{5} * \frac{2}{7} = \frac{6}{35}$$

Properties

Multiplication Property of 0

$$a * 0 = 0 * a = 0$$

Any number times 0 is 0. This is because we are adding that number to itself 0 times.

$$3 * 0 = 0; 0 * 20 = 0$$

Multiplication Property of 1 (Multiplicative Identity)

$$a * 1 = 1 * a = a$$

Any number times 1 is itself. This is because we are adding the number to itself 1 time.

$$4 * 1 = 4; 1 * 2500 = 2500$$

Commutative Property of Multiplication (ordering)

$$a * b = b * a$$

We can change the order in which we multiply 2 numbers.

$$6 * 3 = 18 \quad \text{and} \quad 3 * 6 = 18$$

Associative Property of Multiplication (grouping)

$$(a * b) * c = a * (b * c)$$

The grouping of numbers can be changed without changing the product.

$$5 * (2 * 3) \rightarrow 5 * 6 \rightarrow 30 \quad \text{and} \quad (5 * 2) * 3 \rightarrow 10 * 3 \rightarrow 30$$

Multiplicative Inverse (Reciprocals)

$$a \bullet \frac{1}{a} = \frac{1}{a} \bullet a = 1$$

The product of a number and its multiplicative inverse (reciprocal) is the identity (1).

$$3 * \frac{1}{3} = 1 \quad \text{and} \quad \frac{1}{7} * 7 = 1$$

Distributive Property

Multiplication distributes over addition. That means that the number outside the parenthesis is given to (distributed to) each of the terms inside the parenthesis, using multiplication.

$$\begin{aligned} 5(8 + 2) &\rightarrow 5 * 8 + 5 * 2 \rightarrow 40 + 10 \rightarrow 50 \\ 3(79) &\rightarrow 3(70 + 9) \rightarrow 3 * 70 + 3 * 9 \rightarrow 210 + 27 \rightarrow 237 \\ 3*83 &\rightarrow 3(80 + 3) \rightarrow 3*80 + 3*3 \rightarrow 240 + 9 \rightarrow 249 \end{aligned}$$