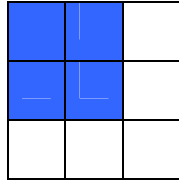# Fractions

A **fraction** is a number of the form $\dfrac{a}{b}$, where *a* and *b* are integers, & *b* is not 0. It represents part of a whole. $\dfrac{a}{b} = \dfrac{\text{part of whole}}{\#\ \text{of parts in the whole}}$

This figure is divided into 9 parts, 4 of which are colored. $\dfrac{4}{9}$ of this figure is colored.

A **proper fraction** is a fraction whose numerator (top) is less than its denominator (bottom), so that its value is less than one. An **improper fraction** is a fraction whose numerator is greater than its denominator, so that its value is greater than one.

If n is any integer, other than 0, then $\dfrac{n}{n} = 1\ \&\ \dfrac{n}{1} = n$

If n is any integer, other than 0, then $\dfrac{0}{n} = 0\ \&\ \dfrac{n}{0} = undefined$

If *a*, *b*, and *c* are numbers, then $\dfrac{a}{b} = \dfrac{a \cdot c}{b \cdot c}\ \&\ \dfrac{a}{b} = \dfrac{a \div c}{b \div c}$ as long as *b* and *c* are not 0. That is, if the numerator and denominator are multiplied or divided by the same nonzero number, the result is an **equivalent fraction**.

Operations on Fractions

Let a, b, c, d be nonzero integers, if zero see appropriate rules above.

Multiplication: $\dfrac{a}{b} \bullet \dfrac{c}{d} = \dfrac{ac}{bd}$

Example:
$\dfrac{3}{5} * \dfrac{2}{7} = \dfrac{6}{35}$

Division(invert & multiply): $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \bullet \dfrac{d}{c} = \dfrac{ad}{bc}$

Example:
$\dfrac{6}{5} \div \dfrac{7}{11} \xrightarrow{Invert\_and\_Multiply} \dfrac{6}{5} * \dfrac{11}{7} \xrightarrow{Multiply} \dfrac{66}{35}$

Addition (Subtraction) *with* common denominators: $\dfrac{a}{b} + \dfrac{c}{b} = \dfrac{a+c}{b}$

Examples:

$$\frac{15}{7} + \frac{1}{7} = \frac{16}{7} \qquad\qquad \frac{17}{7} - \frac{1}{7} = \frac{16}{7}$$

Addition (Subtraction) *without* common denominators: $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad+cb}{bd}$ and then

you MUST reduce OR you can find the LCM for b & d before adding, which is the Least Common Denominator(LCD) for the 2 fractions.

Examples:

$$\frac{2}{3} + \frac{3}{5} \xrightarrow{LCD=3*5=15} \frac{2}{3}*\frac{5}{5} + \frac{3}{5}*\frac{3}{3} \xrightarrow{\text{Multiply Numerators and Denomintors}} \frac{10}{15} + \frac{9}{15} \xrightarrow{\text{Now Add}} \frac{19}{15}$$

$$\frac{2}{3} - \frac{3}{5} \xrightarrow{LCD=3*5=15} \frac{2}{3}*\frac{5}{5} - \frac{3}{5}*\frac{3}{3} \xrightarrow{\text{Multiply Numerators and Denomintors}} \frac{10}{15} - \frac{9}{15} \xrightarrow{\text{Now Subtract}} \frac{1}{15}$$

Mixed Numbers

A mixed number is a whole number added to a fraction, so you can convert to an improper fraction by adding them together. OR you can follow the following steps:

Step 1. Multiply the whole number by the denominator of the fraction.
Step 2. Add the numerator of the fraction to the product from Step 1.
Step 3. Write the sum from Step 2 as the numerator of the improper fraction over the original denominator.

Writing Improper Fractions as Mixed Numbers or Whole Numbers

Step 1. Divide the denominator into the numerator.
Step 2. The whole-number part of the mixed number is the quotient. The fraction part of the mixed number is the remainder over the original denominator.

To perform operation on Mixed numbers, I strongly recommend converting to improper fractions first.

Complex Fractions

A complex fraction is a fraction that contains fractions in the numerator and/or in the denominator.

To simplify a complex fraction
Method 1 (a little bit at a time):
1.) Add or subtract fractions in the numerator or denominator so that the numerator and denominator are each a single fraction.
2.) Perform the indicated division by multiplying the numerator of the complex fraction by the reciprocal of the denominator of the complex fraction.
3.) Write the rational expression in lowest terms.

Example:

$$\frac{\frac{2}{3}+\frac{3}{7}}{\frac{7}{2}-\frac{9}{5}} \xrightarrow[\text{LCD Denominator}=10]{\text{LCD Numerator}=21} \frac{\frac{2}{3}*\frac{7}{7}+\frac{3}{7}*\frac{3}{3}}{\frac{7}{2}*\frac{5}{5}-\frac{9}{5}*\frac{2}{2}} \xrightarrow{Multiply} \frac{\frac{14}{21}+\frac{9}{21}}{\frac{35}{10}-\frac{18}{10}} \xrightarrow[Subtract]{Add} \frac{\frac{23}{21}}{\frac{17}{10}}$$

$$\xrightarrow{Step\,2} \frac{23}{21} \div \frac{17}{10} \xrightarrow{Invert} \frac{23}{21}*\frac{10}{17} \xrightarrow{Multiply} \frac{230}{357}$$

Method 2 (everything at once):
1.) Find the LCD of all the fractions in the complex fraction.
2.) Multiply both the numerator & the denominator of the complex fraction by the LCD found in Step 1.
3.) Perform the indicated operations.
4.) Write the result in lowest terms.

Example:

$$\frac{\frac{5}{3}-2}{\frac{3}{2}+\frac{7}{6}} \xrightarrow{\text{LCD ALL}=6} \frac{\left(\frac{5}{3}-2\right)}{\left(\frac{3}{2}+\frac{7}{6}\right)}*\frac{6}{6} \xrightarrow{Distribute} \frac{\frac{5}{3}*6-2*6}{\frac{3}{2}*6+\frac{7}{6}*6} \xrightarrow{Cancel!} \frac{5*2-2*6}{3*3+7*1}$$

$$\xrightarrow{Multiply} \frac{10-12}{9+7} \xrightarrow[Add]{Subtract} \frac{-2}{2} \xrightarrow{Re\,duce!} -1$$

Use Method 2 if you easily see an LCD for everything. Use Method 1 if you don't.