

Real Number Factoring

A **prime number** is a number whose only factors are 1 & itself (except 1).

Examples: 2, 3, 5, 7, 11, 13

A **composite number** is a natural number which is not a prime number (except 1). A composite number is composed of factors of prime numbers. This means that a composite number can be factored into prime numbers.

Examples: 4, 6, 8, 10, 12, 14, 15

Factors of a composite number are those numbers which when multiplied together produce the composite number. Hence a composite number is a **product** (multiplication) of its factors.

Examples: $36 = 4 \cdot 9$, $12 = 4 \cdot 3$, $6 = 2 \cdot 3$, $25 = 5 \cdot 5$

Prime Factorization is when you write a composite number as a product of its prime factors.

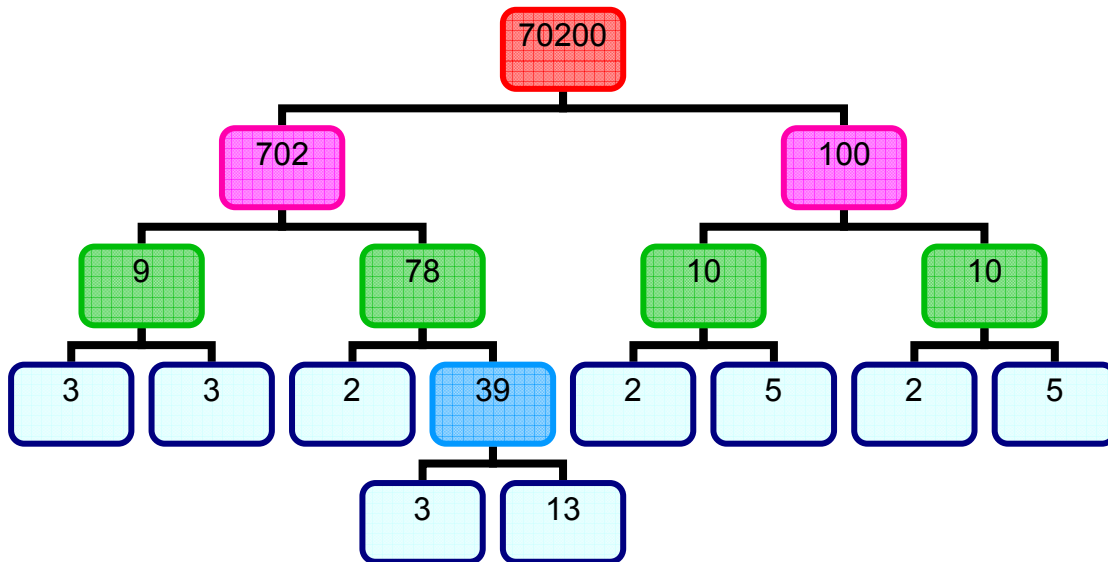
One Method for factoring Real Numbers into their Prime Factorization is the **Factor Tree**. The top of the tree is the number you wish to factor, then each lower branch is a factor of it, continuing down until you reach prime numbers. It is common to circle the end leaf on each branch so you can easily see the prime factors of your original number.

Divisibility Tests: The following can be used to help you factor Real Numbers.

- 2: Any even number, that is, any number that ends in 0, 2, 4, 6, or 8.
- 3: If the sum of its digits is divisible by 3, for example $2346 \rightarrow 2+3+4+6 = 15$, which is divisible by 3, so 2346 is.
- 4: If last 2 digits are divisible by 4, if last 2 are 00, then use last 3 digits to check.
- 5: If it ends in 0 or 5.
- 6: If it is divisible by both 2 AND 3.
- 7: See <http://www.jimloy.com/number/divis.htm>, easier to just divide to check.
- 8: If the last 3 digits are divisible by 8.
- 9: If the sum of its digits is divisible by 9.
- 10: If it ends in 0.
- 11: If the difference between the sum of the odd numbered digits (1st, 3rd, 5th...) and the sum of the even numbered digits (2nd, 4th...) is divisible by 11 (<http://www.jimloy.com/number/divis.htm>). It is easier to just divide to check.

Examples for (Prime) Factoring and Factor Trees:

- $36 = 4 \cdot 9 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$
- 198 (even so we know 2 goes into it; also $1 + 9 + 8 = 18$, so 9 goes into it. I will start factoring with 9, since it is larger and we may get done sooner)
 - $198 = 9 \cdot 22 = 3 \cdot 3 \cdot 2 \cdot 11 = 3^2 \cdot 2 \cdot 11 = 2 \cdot 3^2 \cdot 11$



Since 70200 ends in 2 zeros, we know that 100 is a factor.

Since $7+0+2 = 9$, 9 is a factor of 702.

So $70200 = 3 \cdot 3 \cdot 2 \cdot 3 \cdot 13 \cdot 2 \cdot 5 \cdot 2 \cdot 5 = 2^3 \cdot 3^3 \cdot 5^2 \cdot 13$

Least Common Multiple, LCM, is the smallest number that contains all the factors of the numbers.

Least Common Denominator, LCD, is also called the Least Common Multiple of Denominators, and so is the smallest number that contains all the factors in the collection of denominators.

To find the LCM:

1. Write all the numbers in their prime factorization
2. Write the product that contains each unique factor to the highest power in one number.

Greatest Common Factor, GCF, is the largest number that is a common factor to the list of numbers.

When a list of numbers has only a factor of 1 in common, they are called **relatively prime** (they have no factors in common, 1 being not much use since 1 is the identity).

To find the GCF:

1. Write the numbers in their prime factorization
2. List those factors that are common to each number
3. Multiply the common factors together to get the GCF

Examples:

$$3 = 1 * 3$$

$$4 = 1 * 4 = 2 * 2$$

$$5 = 1 * 5$$

$$12 = 1 * 12 = 2 * 6 = 2^2 * 3 = 4 * 3$$

$$15 = 1 * 15 = 3 * 5$$

LCM/LCD = $3 \cdot 4 \cdot 5 = 60$ (each number has **at most** one 3, one 4, or one 5 as a factor)

GCF = 1 (only factor that is listed in all of the factorings above)

$$30 = 1 * 30 = 3 * 10 = 3 * 2 * 5 = 3 * 2 * 5$$

$$75 = 1 * 75 = 3 * 25 = 3 * 5 * 5 = 3 * 5 * 5$$

$$135 = 1 * 135 = 3 * 45 = 3 * 9 * 5 = 3 * 3 * 3 * 5 = 3 * 3 * 3 * 5$$

LCM/LCD = $2 * 3 * 5 * 5 * 3 * 3 = 2 * 3^3 * 5^2 = 1350$ (30 has a 2, 3 & 5, 75 has a 3 & 2 factors of 5, 135 has 3 factors of 3, and one of 5)

GCF = $3 * 5 = 15$ (they each have **at least** one 3 & at least one 5 as factors)

$$15 = 1 * 15 = 3 * 5$$

$$25 = 1 * 25 = 5 * 5 = 5^2$$

$$27 = 1 * 27 = 3 * 9 = 3 * 3 * 3 = 3^3$$

LCM/LCD = $3^3 * 5^2 = 675$ (3^3 appears in 27 so we must have 3^3 in our LCM, 5^2 appears in 25 so it must be in our LCM, the 3 & 5 from 15 are already listed in the previous two factor sets so they do not need to be repeated.)

GCF = 1 (only factor that is listed in all of the factorings above)

Similarities:

- Write all numbers in Prime Factorization

Differences:

- LCD/LCM you are looking for a number bigger than or equal to the largest number in the list.
- LCD/LCM may be as large as the product of the numbers.
- GCF you are looking a number smaller than or equal to the smallest number in the list.
- GCF may be as small as 1