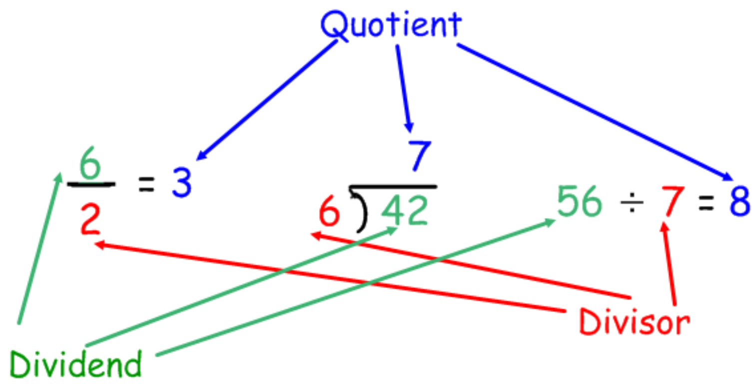
Division of Real Numbers

Just as multiplication can be thought of shorthand notation for addition of whole numbers, division can be thought of shorthand notation for subtraction of whole numbers, which is easily seen in the Long Division Example that follows.



The process of separating a quantity into equal parts is called division. Since division and multiplication are reverse operations, don't forget that a division problem can be checked by multiplying. Be careful the order does matter when dividing!

Whole Numbers

If a & b are whole numbers, and b ≠ 0, then $a \div b = a \times \frac{1}{b}$, where $\frac{1}{b}$ is called the <u>reciprocal</u> of b. This same idea carries through all types of real numbers, just change "whole" to "rational", "real" or "whole numbers" to "integers".

Long Division

$$\begin{array}{r} 579 \\ 423\overline{)244989} \\ \underline{-2115} \\ 3348 \\ \underline{-2961} \\ 3879 \\ \underline{-3807} \\ 72 \end{array}$$

4 goes into 24 - 6 times, 6 x 423 = 2538, which is too big so use 5. 5 x 423 = 2115

Now subtract 2115 from 2449 = 334

Bring down the next digit, the 8.

4 goes into 33 - 8 times, 423 x 8 = 3384, which is too big, so use 7. 423 x 7 = 2961

Now subtract 2961 from 3348 = 387

Bring down the next digit, the 9.

Now subtract 3807 from 3879 = 72

72 is called the remainder. We write the answer as 579 R 72.

Integers

If the 2 numbers have the same sign, the result is positive.
If the 2 numbers have different signs, the result is negative.

Tips:

+number ÷ +number = +number
-number ÷ -number = +number
+number ÷ -number = -number

Examples:

42 ÷ 7 = 6
-42 ÷ -7 = 6
42 ÷ -7 = -6

Decimals

1) Move the decimal point in the divisor to the right until the divisor is a whole number.
2) Move the decimal point in the dividend to the right the *same number of places* as the decimal point was moved in Step 1.
3) Divide. Place the decimal point in the quotient directly over the moved decimal point in the dividend.
    a) If necessary place zeros at the end of the dividend to continue dividing.

Shortcuts with Powers of 10

(10, 100, 1000, …) Move the decimal point to the *left* the same number of places as there are *zeros* in the power of 10.
(.1, .01, .001, …) Move the decimal point to the *right* the same number of places as there are *decimal places* in the power of 10.

Examples:

2.36 ÷ 100 = .0236 156 ÷ .001 = 156000

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           26.17580...
1.24 ) 32.458000
     - 24 8
        7 65
      - 7 44
          218
        - 124
            940
          - 868
              720
                  ...
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Normally you continue with your division until you come to the end, or to the appropriate Significant Digit. In this case we went for a few extra digits, now we will round to the nearest 100^{th}. The quotient is 26.17580…, to the nearest hundredth it is 26.18.

Significant Digit: is the largest of the smallest place values that we know to be accurate, 0 place holders are not included. In this case we had 1.24 & 32.458, the 4 in 1.24 is in the hundredths place value & the 8 in the 32.458 is in the thousandths place value, so the hundredths place value is the significant digit.

Fractions

Invert the second fraction and change the division sign to multiplication, see "Whole Numbers" above, sometimes we just say "invert and multiply".

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$$

Example:

$$\frac{6}{5} \div \frac{7}{11} \xrightarrow{Invert\_and\_Multiply} \frac{6}{5} * \frac{11}{7} \xrightarrow{Multiply} \frac{66}{35}$$

Divisibility Tests

2: Any even number, that is, any number that ends in 0, 2, 4, 6, or 8.
3: If the sum of its digits is divisible by 3, for example 2346 → 2+3+4+6 = 15, which is divisible by 3, so 2346 is.
4: If last 2 digits are divisible by 4, if last 2 are 00, then use last 3 digits to check.
5: If it ends in 0 or 5.
6: If it is divisible by both 2 AND 3.
7: See http://www.jimloy.com/number/divis.htm
8: If the last 3 digits are divisible by 8.
9: If the sum of its digits is divisible by 9.
10: If it ends in 0.
11: If the difference between the sum of the odd numbered digits (1st, 3rd, 5th...) and the sum of the even numbered digits (2nd, 4th...) is divisible by 11 (http://www.jimloy.com/number/divis.htm).

Properties

Division Properties of 0

The quotient of 0 and any number (except 0) is 0 $0 \div a = 0$ or $\frac{0}{a} = 0$

$$\frac{0}{6} = 0, \quad 5\overline{)0}^{\,0}, \quad 0 \div 7 = 0$$

The quotient of any number and 0 is not a number $a \div 0$ is undefined or $\frac{a}{0}$ is undefined

We say that $\frac{6}{0}, \quad 0\overline{)5}, \quad 7 \div 0$ are undefined

Division Properties of 1

The quotient of any number and that same number is 1 $a \div a = 1$ or $\frac{a}{a} = 1$

$$\frac{6}{6} = 1, \quad 5\overline{)5}^{\,1}, \quad 7 \div 7 = 1$$

The quotient of any number and 1 is that same number $a \div 1 = a$ or $\frac{a}{1} = a$

$$\frac{6}{1} = 6, \quad 1\overline{)5}^{\,5}, \quad 7 \div 1 = 1$$