Exponents & Radical Rules

$\sqrt[n]{a} = a^{\frac{1}{n}}$, whenever $\sqrt[n]{a}$ is a real number. Since a radical is another way of writing exponents, radicals follow all the same rules as exponents, so long as the radical expression describes a real number. Sometimes it is easier to work with a radical if it is first written as an exponent.

Let a, b be non-zero real numbers (or variables), r & s be positive integers, and m, n & p be rational numbers such that a^p, b^p, a^{mp}, a^n, b^{np}, b^m are Real Numbers! If they are not, we will cover later with Complex Numbers.

$a^{\frac{r}{s}} = \left(a^r\right)^{\frac{1}{s}} = \left(a^{\frac{1}{s}}\right)^r = \left(\sqrt[s]{a}\right)^r = \sqrt[s]{a^r}$, provided $a^{\frac{1}{s}}$ & $\left(a^r\right)^{\frac{1}{s}}$ are real numbers. This comes from the fact that $\frac{r}{s} = \frac{r}{1} \bullet \frac{1}{s}$ and the Power Rule for Exponents.

Product Rule: $a^m \cdot a^n = a^{m+n}$

Power Rule: $\left(a^m\right)^n = a^{mn}$

Power of a Product Rule: $(ab)^p = a^p b^p$ or $(a^m b^n)^p = a^{mp} b^{np}$

Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$

Power of a Quotient Rule: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ Zero Exponent: $a^0 = 1$, $a \neq 0$

Negative Exponents: $a^{-n} = \frac{1}{a^n}$, $a \neq 0$ & $\left(\frac{a}{b}\right)^{-r} = \frac{b^r}{a^r}$

Examples:

- $- 256^{1/2} \rightarrow - (256^{1/2}) \rightarrow - (16) \rightarrow - 16$
- $(- 256)^{1/2} \rightarrow$ is not a real number. Order of Operations still holds!
- $- 216^{1/3} \rightarrow - (216^{1/3}) \rightarrow - (6) \rightarrow - 6$
- $(- 216)^{1/3} \rightarrow - 6$, since we are looking for an odd root not an even root.
- $$(8)^{\frac{2}{3}} + \sqrt{25} \rightarrow \left((8)^{\frac{1}{3}}\right)^2 + 5 \rightarrow (2)^2 + 5 \rightarrow 4 + 5 \rightarrow 9$$

$$\left(\frac{16x^4}{81y^5}\right)^{-\frac{3}{4}} \xrightarrow[\text{exponent\_first}]{\text{remove\_negative}} \left(\left(\frac{16x^4}{81y^5}\right)^{-1}\right)^{\frac{3}{4}} \rightarrow \left(\left(\frac{81y^5}{16x^4}\right)\right)^{\frac{3}{4}} \xrightarrow[\text{root\_helps\_next}]{\text{normally\_taking}}$$

- $$\left(\frac{81y^5}{16x^4}\right)^{\frac{1}{4}\bullet 3} \rightarrow \left(\left(\frac{81y^5}{16x^4}\right)^{\frac{1}{4}}\right)^3 \rightarrow \left(\left(\frac{3^4 y^5}{2^4 x^4}\right)^{\frac{1}{4}}\right)^3 \xrightarrow{PowerRule\frac{1}{4}} \left(\left(\frac{(3^4)^{\frac{1}{4}}(y^5)^{\frac{1}{4}}}{(2^4)^{\frac{1}{4}}(x^4)^{\frac{1}{4}}}\right)\right)^3$$

$$\xrightarrow{4\bullet\frac{1}{4}=1} \left(\frac{3y^{\frac{5}{4}}}{2x}\right)^3 \xrightarrow{PowerRule} \frac{(3)^3\left(y^{\frac{5}{4}}\right)^3}{(2x)^3} \xrightarrow{Evaluate} \frac{27y^{\frac{15}{4}}}{8x^3}$$

$$\sqrt{\sqrt[6]{x^3}} \xrightarrow{\text{rewrite\_as\_exponents}} \left(\sqrt[6]{x^3}\right)^{\frac{1}{2}} \xrightarrow{\text{one\_at\_a\_time}} \left((x^3)^{\frac{1}{6}}\right)^{\frac{1}{2}} \xrightarrow{PowerRule}$$

- $$\left(x^{3\bullet\frac{1}{6}}\right)^{\frac{1}{2}} \rightarrow \left(x^{\frac{1}{2}}\right)^{\frac{1}{2}} \xrightarrow{PowerRule} x^{\frac{1}{2}\bullet\frac{1}{2}} \rightarrow x^{\frac{1}{4}} \rightarrow \sqrt[4]{x}$$
