Properties of Radicals

The following properties are used for simplifying radicals and radical expressions. If you need to perform operations on radical expressions it is much easier to work with them if they are simplified.

Multiplication Property of Radicals: $\sqrt[r]{a} \cdot \sqrt[r]{b} = \sqrt[r]{ab}$, provided $\sqrt[r]{a}$ & $\sqrt[r]{b}$ are real numbers

Division Property of Radicals: $\sqrt[r]{\dfrac{a}{b}} = \dfrac{\sqrt[r]{a}}{\sqrt[r]{b}}$, provided $\sqrt[r]{a}$ & $\sqrt[r]{b}$ are real numbers, and $\sqrt[r]{b} \neq 0$

Examples:

$\blacktriangleright \quad 5\sqrt{75} \xrightarrow{Multiplica tion\_\Pr operty} 5\sqrt{25 \bullet 3} \to 5\sqrt{25} \bullet \sqrt{3} \to 5 \bullet 5\sqrt{3} \to 25\sqrt{3}$

$\sqrt[3]{\dfrac{81x^6 y^3}{27x^5 z}} \xrightarrow{Division\_\Pr operty} \dfrac{\sqrt[3]{81x^6 y^3}}{\sqrt[3]{27x^5 z}} \xrightarrow{Multiplication\_\Pr operty} \dfrac{\sqrt[3]{81} \bullet \sqrt[3]{x^6} \bullet \sqrt[3]{y^3}}{\sqrt[3]{27} \bullet \sqrt[3]{x^5} \bullet \sqrt[3]{z}}$

$\blacktriangleright \quad \dfrac{\sqrt[3]{3^4} \bullet \sqrt[3]{(x)^{3+3}} \bullet y}{\sqrt[3]{3^3} \bullet \sqrt[3]{(x)^{3+2}} \bullet \sqrt[3]{z}} \to \dfrac{\sqrt[3]{3^{3+1}} \bullet \sqrt[3]{x^3 \bullet x^3} \bullet y}{3 \bullet \sqrt[3]{x^3 \bullet x^2} \bullet \sqrt[3]{z}} \to \dfrac{\sqrt[3]{3^3 \bullet 3} \bullet \sqrt[3]{x^3} \bullet \sqrt[3]{x^3} \bullet y}{3 \bullet \sqrt[3]{x^3} \bullet \sqrt[3]{x^2} \bullet \sqrt[3]{z}} \to$

$\dfrac{3\sqrt[3]{3} \bullet x \bullet x \bullet y}{3 \bullet x \bullet \sqrt[3]{x^2} \bullet \sqrt[3]{z}} \xrightarrow{Cancel\_as\_appropriate} \dfrac{\sqrt[3]{3} \bullet x \bullet y}{\sqrt[3]{x^2} \bullet \sqrt[3]{z}} \xrightarrow{\operatorname{Re} write} \dfrac{xy\sqrt[3]{3}}{\sqrt[3]{x^2 z}}$