Introduction to Radical Expressions

Square Root

The radical symbol, $\sqrt{}$, is used to denote the square root of what is inside. $\sqrt{4} = 2$ means that when 2 is squared we get 4. When no sign, + or -, is in front of the radical we want the positive root. This positive square root is called the **principal square root**.
A **perfect square** is a number whose square root is a rational number.

Examples:

$\sqrt{16} = 4$ $\quad -\sqrt{25} = -5$ $\quad \sqrt{-81}$ is not a real number $\quad \sqrt{\frac{4}{144}} \to \frac{2}{12} \to \frac{1}{6}$

Other Radicals

The symbol $\sqrt[n]{a}$ is also called a radical sign, we want to know what number raised to the n^{th} power will give us a, or we can say "what is the n^{th} root of a", for some whole number n, and some real number a. n is called the **<u>index</u>** of the radical, a is called the **<u>radicand</u>**.
Warning! Do not confuse the radical, $\sqrt{}$, with the long division symbol, $\overline{)}$. They are different!

Tips

- If n is even & there is a negative under the radical the result is **not** a real number.
- If n is odd then there is only one solution.
- $\sqrt[n]{a^n} = |a|$, when n is an even positive integer, and a is positive. This is called the **<u>principal *n*th root</u>**.
- $\sqrt[n]{a^n} = a$, when n is an odd positive integer, and a is any real number.
- When solving applications and we take the square root, we need both unless one of the answers does not make sense in the problem. Remember negative time and negative length do not make sense.

Examples:

$\sqrt[3]{125} = 5$ $\quad \sqrt[5]{32} = 2$ $\quad \sqrt[4]{1296} = 6$ $\quad \sqrt[8]{-65536}$ is not a real number $\quad \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

Radical Expressions

$\sqrt{3}$ $2x\sqrt{3y}$ $2x+3\sqrt{2}$ $4\sqrt[5]{27x^4}$

When working with radical expressions, just like working with any other variable expression, it is often best to simplify it first.

Simplifying radical expressions means to get as much out from under the radical as possible. Just like what was done in the examples above with numbers.

Examples:

$\sqrt{x^4} = \left|x^2\right| \xrightarrow{always\_positive} x^2$ $\sqrt{4x^2} = 2|x|$ $\sqrt[3]{216y^9} = 6y^3$

Recall that rational expressions had problems when the denominator was zero, we have a similar situation when the index of the radical is even. If the radicand is negative and the index is even then the expression is not a real number: $\sqrt{-81}$ & $\sqrt[8]{-65536}$ are not a real numbers. Thus if we have variables under the radical it is necessary to restrict the values of the variables. When working with a radical function we restrict the domain so that the expression under the radical is greater than or equal to zero.

Examples:

$\sqrt{2x+3}$ set 2x + 3 ≥ 0 & solve for x. 2x ≥ -3 → $x \geq -\frac{3}{2}$

So $\sqrt{2x+3}$ is a real number provided $x \geq -\frac{3}{2}$.

$f(x) = \sqrt[4]{x+7}$; x + 7 ≥ 0 → x ≥ -7

So the domain of $f(x) = \sqrt[4]{x+7}$ is $[-7, \infty)$

For powers up to 10^{10} see the Completed Exponent Table in the Prerequisites Folder.