Radical Expressions
Division

A radical expression of the form $a-b\sqrt{c}$ is **the conjugate** of $a+b\sqrt{c}$. Notice that when we multiply conjugates, it is similar to the Special Products formula for sum & difference.

Example:

$$\left(2-\sqrt{x}\right)\left(2+\sqrt{x}\right)\xrightarrow{FOIL}2*2+2\sqrt{x}-2\sqrt{x}-\sqrt{x}*\sqrt{x}\xrightarrow{Simplify}4-\sqrt{x*x}\rightarrow 4-x$$

Notice that this has removed all radicals from the expression, this changes the expression.

Rationalization (division)

To remove a radical from the denominator, multiply the fraction by the appropriate "1".

a) If the denominator is just 1 term with a radical, use a radical with the same index and smallest possible radicand.
   i) If the denominator is $\sqrt{3}$ use $\frac{\sqrt{3}}{\sqrt{3}}$
   ii) If the denominator is $\sqrt[3]{2^2}$ use $\frac{\sqrt[3]{2}}{\sqrt[3]{2}}$
b) If the denominator has two terms, use the conjugate of the denominator
   i) If the denominator is $3+\sqrt{2}$ use $\frac{3-\sqrt{2}}{3-\sqrt{2}}$.

Examples:

$$\frac{3x}{\sqrt{6}}\xrightarrow{use\sqrt{6}}\frac{3x}{\sqrt{6}}*\frac{\sqrt{6}}{\sqrt{6}}\rightarrow\frac{3x\sqrt{6}}{6}$$

$$\frac{3}{\sqrt[3]{x^2}}\xrightarrow{use\sqrt[3]{x}}\frac{3}{\sqrt[3]{x^2}}*\frac{\sqrt[3]{x}}{\sqrt[3]{x}}\rightarrow\frac{3\sqrt[3]{x}}{\sqrt[3]{x^2*x}}\rightarrow\frac{3\sqrt[3]{x}}{x}$$

$$\frac{x}{3+\sqrt{x}}\xrightarrow{use\left(3-\sqrt{x}\right)}\frac{x}{3+\sqrt{x}}*\frac{3-\sqrt{x}}{3-\sqrt{x}}\rightarrow\frac{x\left(3-\sqrt{x}\right)}{\left(3+\sqrt{x}\right)\left(3-\sqrt{x}\right)}\xrightarrow{Multiply}$$

$$\frac{3x-x\sqrt{x}}{3*3-3\sqrt{x}+3\sqrt{x}-\sqrt{x}*\sqrt{x}}\rightarrow\frac{3x-x\sqrt{x}}{9-x}$$

$$\frac{3x-2}{\sqrt{2}-\sqrt{x}} \xrightarrow{use\sqrt{2}+\sqrt{x}} \frac{3x-2}{\sqrt{2}-\sqrt{x}} * \frac{\sqrt{2}+\sqrt{x}}{\sqrt{2}+\sqrt{x}} \rightarrow \frac{(3x-2)(\sqrt{2}+\sqrt{x})}{(\sqrt{2}-\sqrt{x})(\sqrt{2}+\sqrt{x})} \xrightarrow{FOIL}$$

$$\frac{3x\sqrt{2}+3x\sqrt{x}-2\sqrt{2}-2\sqrt{x}}{\sqrt{2}*\sqrt{2}+\sqrt{2}*\sqrt{x}-\sqrt{x}*\sqrt{2}-\sqrt{x}*\sqrt{x}} \xrightarrow{Top\_is\_done} \frac{3x\sqrt{2}+3x\sqrt{x}-2\sqrt{2}-2\sqrt{x}}{2+\sqrt{2}*\sqrt{x}-\sqrt{2}*\sqrt{x}-x} \rightarrow$$

$$\frac{3x\sqrt{2}+3x\sqrt{x}-2\sqrt{2}-2\sqrt{x}}{2-x}$$

Note: Yes, sometimes the "simplified" expression looks more complicated than the original, but it no longer has radicals in the denominator.

A Radical Expression is Simplified When:

1.) The radicand contains no factor greater than 1 that is a perfect power of the index.
2.) There is no fraction under the radical.
3.) There is no radical in the denominator of an expression.
4.) Any exponents under the radical & the index of the radical are relatively prime (no factors in common except 1)