Exponents

Let a, b be a real numbers (or variables), m & n real numbers (they will normally be integers, in CH 1, or rational numbers, in CH 6)

a^n = a * a * a * a * … * a, n times

a is called the **base** & n is the **exponent** (power or degree)

Rules

Product Rule: $a^m * a^n = a^{m+n}$

Power Rule: $\left(a^m\right)^n = a^{mn}$

Power of a Product Rule: $(ab)^p = a^p b^p$ or $(a^m b^n)^p = a^{mp} b^{np}$

Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$

Power of a Quotient Rule: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Zero Exponent: $a^0 = 1$, $a \neq 0$

Negative Exponents: $a^{-n} = \frac{1}{a^n}$, $a \neq 0$

All of the above rules come directly from the definition of an exponent.

Example 1:

$(2x)^2(4x)^3$ → using the power of a product rule → $2^2x^2 4^3x^3$ → $4x^2 64x^3$ → commutative property of multiplication → $4*64x^2x^3$ → the product rule → $256x^{2+3}$ → $256x^5$

Example 2:

$$\left(\frac{2x}{y^2}\right)^3\left(\frac{y^2}{x^4}\right)^{-1} \xrightarrow[\text{using the power of a quotient rule}]{\text{1st remove parentheses}} \frac{(2x)^3}{\left(y^2\right)^3} \bullet \frac{\left(y^2\right)^{-1}}{\left(x^4\right)^{-1}} \xrightarrow[\text{using the power or power of a product rule}]{\text{then remove parentheses}} \frac{2^3x^3}{y^6} \bullet \frac{y^{-2}}{x^{-4}}$$

$$\xrightarrow[\text{use negative exponent rule}]{\text{recommended next step}} \frac{2^3x^3}{y^6} \bullet \frac{x^4}{y^2} \xrightarrow{\text{finally the product rule}} \frac{8x^7}{y^8}$$

In any one problem we can use several of the rules for exponents. Since exponents can be considered short hand for multiplication, the order in which we apply the rules is convenience. Though removing parentheses is a very good place to start, if we cannot simplify what we have first. We will be using these rules in various situations through out the course, most especially when we work with polynomials and radical expressions.