

Complex Numbers

Definitions

- ◇ $i^2 = -1$, so $i = \sqrt{-1}$. Thus $\sqrt{-b} = i\sqrt{b}$, if b is a positive number.
- ◇ An **imaginary number** is a number which is the square root of a negative number.
- ◇ A number of the form " $a + bi$ " is called a **complex number**, where a & b are real numbers. We call **a** the real part & **b** the imaginary part.
- ◇ Note the set of Real Numbers are a subset of the Complex Numbers.
- ◇ The complex number $a - bi$ is called the **complex conjugate** of $a + bi$.
- ◇ Two complex numbers are **equal** if the real parts and the imaginary parts are equal.

Examples:

$$\sqrt{-9} = 3i$$

$3 + 2i$, $4 + 0i = 4$ both are Complex Numbers.

$4 + 3i$ & $4 - 3i$ are complex conjugates.

$a + bi = 3 - 4i$ if $a = 3$ and $b = -4$

$$i^3 = i^{2+1} = i^2 i = (-1)i = -i$$

$$i^{14} = i^{2 \cdot 7} = (-1)^7 = -1$$

One of the reasons for studying complex numbers is when solving some polynomials they come up as the solution.

Conjugate Pairs Theorem: If $P(x) = 0$ is a polynomial equation with real or complex coefficients and the complex number $a + bi$ ($b \neq 0$) is a root, then so is $a - bi$.

NOTES:

- When solving polynomials complex numbers come in pairs.
- Linear Equations cannot have a complex root.
- Quadratic have 2 complex roots, 2 real roots or one double real root, since a quadratic must have 2 roots.

We will see this when we solve Quadratic Equations with methods other than Factoring.