LCD/LCM, Adding & Subtracting Rational Expressions

The **Least Common Denominator** (LCD), or **Least Common Multiple** (LCM) of the Denominators, is the simplest polynomial that contains the factors of the polynomials from the denominators of the rational expressions.

Finding the Least Common Denominator of Two Rational Expressions (same as for fractions):

1.) Factor the denominators completely.
2.) The LCD is the product of all the unique factors raised to their highest powers.

Example 1:

Find the LCD of $\frac{8x^2}{(x-6)^2}$ & $\frac{13x}{5x-30}$

1.) Factor denominators
   a. $(x - 6)^2$ is factored, just remember there are 2 of the same factor. They must both be in the LCD.
   b. 5x – 30 → 5(x – 6)
2.) LCD: $5(x - 6)^2$ lists each unique factor to its highest power in a denominator.

Example 2:

Find LCD of $\frac{2x+5}{3x-7}$ & $\frac{5}{7-3x}$ The denominators look similar, so the first thing you should do is see if a factor of (– 1) will change one to the other. (– 1)(7 – 3x)→ – 7 + 3x → 3x – 7, which is the first denominator. So the LCD is (– 1)(7 – 3x) or (– 1)(3x – 7)! In this case as long as you multiply one of the denominators by (– 1) you will get the other.

To Add or Subtract Rational Expressions

With Common Denominators

1.) Add or Subtract Numerators
2.) Place on top of the Common Denominator.
3.) Simplify if possible.

Example 1:

$$\frac{9}{3+y}+\frac{y+1}{3+y}\rightarrow\frac{(9)+(y+1)}{3+y}\rightarrow\frac{9+y+1}{3+y}\rightarrow\frac{y+10}{3+y}$$

Example 2:

$$\frac{x^2+9x}{x+7}-\frac{4x+14}{x+7}\longrightarrow\frac{(x^2+9x)-(4x+14)}{x+7}\xrightarrow{Distribute(-)}$$

$$\frac{x^2+9x-4x-14}{x+7}\xrightarrow{Combine\_Like\_Terms}\frac{x^2+5x-14}{x+7}$$

$$\xrightarrow[\text{Use Denominator For Hint}]{factor}\frac{(x+7)(x-2)}{x+7}\xrightarrow{Cancel}x-2$$

With Not Common Denominators

1.) Find the LCD
2.) Rewrite each expression with LCD
3.) Add or Subtract as above
4.) Simplify if possible

Examples:

$$\frac{-8}{x^2-1}-\frac{7}{1-x^2}$$

1.) Find the LCD.
   a. Since the 2 denominators are similar, try the "-1 trick".
      i. $x^2 - 1 = (x + 1)(x - 1) = (-1)(1 - x)(1 + x) = 1 - x^2$
      ii. To keep everything in Standard Form, I will use (x + 1)(x – 1) as my LCD, which means the second term needs to be multiplied (top & bottom) by (-1).

2.) $\frac{-8}{x^2-1}-\frac{(-1)7}{(-1)(1-x^2)}\longrightarrow\frac{-8}{x^2-1}-\frac{-7}{x^2-1}$

3.) $\frac{-8}{x^2-1}-\frac{-7}{x^2-1}\rightarrow\frac{(-8)-(-7)}{x^2-1}\rightarrow\frac{-8+7}{x^2-1}\rightarrow\frac{-1}{x^2-1}$

$$\frac{15}{2x-4}+\frac{x}{x^2-4}\xrightarrow{\text{Factor Denominato rs}}\frac{15}{2(x-2)}+\frac{x}{(x+2)(x-2)}$$

$$\xrightarrow{LCD=2(x-2)(x+2)}\frac{15}{2(x-2)}*\frac{(x+2)}{(x+2)}+\frac{x}{(x+2)(x-2)}*\frac{2}{2}\rightarrow$$

$$\frac{15(x+2)+2x}{2(x+2)(x-2)}\rightarrow\frac{15x+30+2x}{2(x+2)(x-2)}\rightarrow\frac{17x+30}{2(x-2)(x+2)}$$

$$\frac{2x-5}{6x+9}-\frac{4}{2x^2+3x}\xrightarrow{\text{Factor Denominators}}\frac{2x-5}{3(2x+3)}-\frac{4}{x(2x+3)}\xrightarrow{LCD=3x(2x+3)}$$

$$\frac{2x-5}{3(2x+3)}*\frac{x}{x}-\frac{4}{x(2x+3)}*\frac{3}{3}\rightarrow\frac{(2x-5)x}{3x(2x+3)}-\frac{4*3}{3x(2x+3)}$$

$$\rightarrow\frac{2x^2-5x-12}{3x(2x+3)}\xrightarrow{Factor}\frac{(2x+3)(x-4)}{3x(2x+3)}\xrightarrow{Cancel}\frac{x-4}{3x}$$