Radical Equations

If two numbers are equal and they are raised to the same power then they are still equal. $a = b, \text{ then } a^n = b^n$. Similarly, if two expressions are equal and they are raised to the same power then they are still equal, whether to an integer or a rational power.

An **extraneous solution** is a solution to the equation when it has been raised to a power, but not a solution to the original equation. This occurs because we are adding roots to the equation. If you remember when solving polynomial equations, you have the same number of roots as the degree of the polynomial.

HINT: negative under a radical with an even index is not a real number.

Think about this, when you take a real number to an exponent, you are multiplying that number by itself the number of times as the exponent. So if the exponent is an even number, you will have an even number of factors. If you have an even number of negative signs in a multiplication, you ALWAYS have a positive answer when working with Real Numbers.

To solve Radical Equations

1.) Move all terms containing radicals to one side of the equation & everything else to the other side.
    a. If there is more than 1 unique radical, only isolate one radical at a time.
    b. Make sure there is no coefficient for the radical.
2.) Raise both sides of the equation to the power of the index.
    a. If there is another radical go back to step 1, repeat until no more radicals.
3.) Solve for the variable.
4.) Check the values in the original equation. Some of them may not work.

Example 1: Solve: $9-\sqrt{4k+1}=0$	Check 20 = k
$9-\sqrt{4k+1}=0$ $9=\sqrt{4k+1}$ $(9)^2=\left(\sqrt{4k+1}\right)^2$ $81=4k+1$ $80=4k$ $\frac{80}{4}=\frac{4k}{4}\rightarrow 20=k$	$9-\sqrt{4(20)+1}\stackrel{?}{=}0$ $9-\sqrt{81}$ $9-9$ $0=0$
So k = 20 is the solution to the equation.	

Example 2: Solve: $\sqrt{3x-2}-\sqrt{x+3}=1$	Check x = 1
$\sqrt{3x-2}-\sqrt{x+3}=1$ $\sqrt{3x-2}=1+\sqrt{x+3}$ $\left(\sqrt{3x-2}\right)^2=\left(1+\sqrt{x+3}\right)^2$ $3x-2=1+2\sqrt{x+3}+x+3$ $-2\sqrt{x+3}=-3x+2+1+x+3$ $-2\sqrt{x+3}=-2x+6$ $\frac{-2\sqrt{x+3}}{-2}=\frac{-2x+6}{-2}$ $\sqrt{x+3}=x-3$ $\left(\sqrt{x+3}\right)^2=(x-3)^2$ $x+3=x^2-6x+9$ $0=x^2-7x+6$ $0=(x-1)(x-6)$ $0=x-1\ \&\ 0=x-6$ $1=x\ \&\ 6=x$	$\sqrt{3(1)-2}-\sqrt{(1)+3}\stackrel{?}{=}1$ $\sqrt{3-2}-\sqrt{1+3}$ $\sqrt{1}-\sqrt{4}$ $1-2$ $-1\neq 1$ So x = 1 is not a solution. **Check x = 6** $\sqrt{3(6)-2}-\sqrt{(6)+3}\stackrel{?}{=}1$ $\sqrt{18-2}-\sqrt{9}$ $\sqrt{16}-3$ $4-3$ $1=1$ So x = 6 is a solution.