Quadratic in Form

Equations that are **Quadratic in Form** are any equation that can be written as $au^2 + bu + c = 0$, where u is a variable expression.

Examples

$3(2x + 5)^2 + 7(2x + 5) + 2 = 0$ Let $u = 2x + 5$

$3u^2 + 7u + 2 = 0$

$(3u + 1)(u + 2) = 0$ Set each linear factor equal to zero.

$3u + 1 = 0$ $u + 2 = 0$

$u = -\frac{1}{3}$ $u = -2$ Substitute back in $u = 2x + 5$

$-\frac{1}{3} = 2x + 5 \to -\frac{16}{3} = 2x \to -\frac{8}{3} = x$

Or

$-2 = 2x + 5 \to -7 = 2x \to -\frac{7}{2} = x$

$x = -\frac{8}{3}$ and $x = -\frac{7}{2}$.

$x^4 + 10 = 7x^2$

$x^4 - 7x^2 + 10 = 0$ Let $u = x^2$

$u^2 - 7u + 10 = 0$

$(u - 5)(u - 2) = 0$ Set each linear factor equal to zero.

$u - 5 = 0$ $u - 2 = 0$

$u = 5$ $u = 2$ Substitute back in $u = x^2$

$x^2 = 5$ $x^2 = 2$ Take the square root of both sides.

$\sqrt{x^2} = \pm\sqrt{5}$ $\sqrt{x^2} = \pm\sqrt{2}$

$x = \pm\sqrt{2}, x = \pm\sqrt{5}$

$\left(\frac{b-5}{6}\right)^2 - \left(\frac{b-5}{6}\right) - 6 = 0$ Let $u = \frac{b-5}{6}$

$u^2 - u - 6 = 0$

$(u - 3)(u + 2) = 0$ Set each linear factor equal to zero.

$u - 3 = 0$ $u + 2 = 0$

$u = 3$ $u = -2$ Substitute back in $u = \frac{b-5}{6}$

$\frac{b-5}{6} = 3$ $\frac{b-5}{6} = -2$

$b - 5 = 18$ $b - 5 = -12$

$b = 23$ $b = -7$

$x^{\frac{1}{2}} - 3x^{\frac{1}{4}} + 2 = 0$ Let $u = x^{\frac{1}{4}}$, because $\frac{1}{4} * 2 = \frac{1}{2}$, remembering exponent rules.

$u^2 - 3u + 2 = 0$

$(u - 1)(u - 2) = 0$ Set each linear factor equal to zero.

$u - 1 = 0$ $u - 2 = 0$

$u = 1$ $u = 2$ Substitute back in $u = x^{\frac{1}{4}}$

$x^{\frac{1}{4}} = 1$ $x^{\frac{1}{4}} = 2$ Raise both side to the 4^{th} power.

$x = 1^4$ $x = 2^4$

$x = 1$ $x = 16$

$(2y + 4)^2 = 8y + 23$ Let u = 2y + 4, since that is squared. Change form.

$(2y + 4)^2 = 8y + 16 + 7$ → $(2y + 4)^2 = 4(2y + 4) + 7$

$u^2 = 4u + 7$

$u^2 - 4u - 7 = 0$ This does not factor, use Quadratic Formula.

$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ a = 1, b = -4, c = -7

$u = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-7)}}{2(1)}$ Simplify.

$u = \frac{4 \pm \sqrt{16 + 28}}{2}$

$u = \frac{4 \pm \sqrt{44}}{2}$

$u = \frac{4 \pm 2\sqrt{11}}{2}$ Reduce.

$u = 2 \pm \sqrt{11}$