Quadratic Equations

A Quadratic Equation in **standard form** is $ax^2 + bx + c = 0$, where a, b, and c are Real Numbers, with $a \neq 0$.

The number and type of solutions for a quadratic equation will vary based on the **discriminant**, $b^2 - 4ac$. Remember the equation MUST be in standard form!

1.) If $b^2 - 4ac = 0$, there will be **one unique solution**, it is called a double root.

   a. The quadratic equation will factor as a squared binomial.

2.) If $b^2 - 4ac > 0$, and is a perfect square, there will be **two unique rational solutions.**

   a. The quadratic equation might factor with integers.

3.) If $b^2 - 4ac > 0$, but not a perfect square, there will be **two unique real solutions**.

   a. The quadratic equation will not factor with integers.

4.) If $b^2 - 4ac < 0$, there will be **two unique complex solutions** & they will be complex conjugates.

   a. The quadratic equation will not factor.

Examples:

- $6x^2 - 13x + 2 = 0 \rightarrow a = 6, b = -13, c = 2$
  - Discriminant = $b^2 - 4ac \rightarrow (-13)^2 - 4(6)(2) \rightarrow 169 - 48 \rightarrow 121 \rightarrow (11)^2$, this fits case 2, so there are 2 rational solutions.

- $x^2 + 4x - 29 = 0 \rightarrow a = 1, b = 4, c = -29 \rightarrow$
  - Discriminant = $b^2 - 4ac \rightarrow (4)^2 - 4(1)(-29) \rightarrow 16 + 116 \rightarrow 132$, this fits case 3, so there are 2 real solutions.

- $x^2 + 2x + 1 = 0 \rightarrow a = 1, b = 2, c = 1$
  - Discriminant = $b^2 - 4ac \rightarrow (2)^2 - 4(1)(1) \rightarrow 4 - 4 \rightarrow 0$, this fits case 1, so there is 1 unique solution.

- $x^2 + 2x + 5 = 0 \rightarrow a = 1, b = 2, c = 5$
  - Discriminant = $b^2 - 4ac \rightarrow (2)^2 - 4(1)(5) \rightarrow 4 - 20 \rightarrow -16$, this fits case 4, so there are 2 complex solutions.