Methods for Solving Quadratic Equations

Complete the Square

1.) Write the equation with the x terms on one side & the constant term on the other, $ax^2 + bx = -c$

2.) Divide through by a, $x^2 + \frac{b}{a}x = -\frac{c}{a}$

3.) Add $\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2$ to both sides of the equation to complete the square

4.) Now you can factor the left side using the perfect square trinomial formula (see Special Factoring withPolynomials), $\left(x+\frac{b}{2a}\right)^2 = -\frac{c}{a}+\left(\frac{b}{2a}\right)^2$

5.) Take the square root of both sides

6.) Solve for x

7.) Check solutions, some may not work…

Examples:

1.) $y^2 + 20y = 0 \rightarrow a = 1, b = 20$

2.) $\left(\frac{1}{2}\bullet\frac{b}{a}\right)^2 \rightarrow \left(\frac{1}{2}\bullet\frac{20}{1}\right)^2 \rightarrow (10)^2$

3.) add $(10)^2$ to both sides of the equation $\rightarrow y^2 + 20y + (10)^2 = (10)^2$ (Note, I left it in the form I did to remind me that I now have a perfect square trinomial on the left.) $\rightarrow$

4.) $(y + 10)^2 = (10)^2 \rightarrow$

5.) $\sqrt{(y+10)^2} = \pm\sqrt{(10)^2} \rightarrow y+10 = \pm 10 \rightarrow$

6.) $y = -10 \pm 10 \rightarrow y = 0 \;\&\; y = -20$

1.) $m^2 - 12m + 33 = 0 \rightarrow m^2 - 12m = -33 \rightarrow$

2.) $\left(\frac{1}{2} \bullet \frac{b}{a}\right)^2 \rightarrow \left(\frac{1}{2} \bullet \frac{-12}{1}\right)^2 \rightarrow (-6)^2$

3.) add $(-6)^2$ to both sides of the equation, I leave in the (-) to remind me that is what I need since the b term is negative.

4.) $m^2 - 12m + (-6)^2 = -33 + (-6)^2 \rightarrow (m-6)^2 = -33 + 36 \rightarrow$

5.) $(m-6)^2 = 3 \rightarrow \sqrt{(m-6)^2} = \pm\sqrt{3} \rightarrow$

6.) $m - 6 = \pm\sqrt{3} \rightarrow m = 6 \pm \sqrt{3}$

From the Complete the Square Method Step 4 above, we get the Quadratic Formula:

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \xrightarrow{\text{Add fractions on Right Hand Side}} \left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\xrightarrow{\text{Now take Square Root of both sides}} x + \frac{b}{2a} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}} \xrightarrow[\text{And Rationalize}]{\text{Now x term by itself}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \xrightarrow{Combine} x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$