Linear Equations in 1 Variable

Definitions

An **equation** is an expression that contains equality. It is a sentence stating an equality, expression 1 = expression 2. My name is Tamara, is means equals…

A **first degree equation in one variable** is an equation that contains only one variable, which is not below a fraction line and it only has an exponent of 1.

A **linear equation** is a first degree equation in 1 or more variables. In 1 variable the Standard Form is $ax + b = 0$, where a and b are real numbers and $a \neq 0$.

To **solve** an equation we find the value(s) for the variable(s) that makes the equation true. These values are called **solutions** or **roots**. The **Solution Set** is the set of all solutions to the equation.

- An equation is said to have **no solution** when no real number will work in place of the variable. The equation is called inconsistent. The solution set here is the empty set. { }, Ø

- An equation is called a **conditional equation** if only one solution will work. The solution set for this equation contains only one real number. {-4}

- An equation is called an **identity** if any real number will work. The solution set for this type of equation contains all real numbers. $\{-\infty, \infty\}$

Properties of Equality

Addition (Subtraction): if a = b, then a + c = b + c (a = b, then a – c = b – c)

Multiplication (Division): if a = b, then ac = bc (a = b, then a ÷ c = b ÷ c), so long as $c \neq 0$

Both of these properties mean, whatever we do to one side of the equation, we must also do to the other side. When solving an equation we will be doing the operation that undoes the operation in the original equation, for example x + 4 = 12, the operation is addition, to undo this we subtract 4 from both sides.

Steps for Solving Linear Equations

1) If an equation involves a fraction or a decimal
   a. Fractions: multiply all terms on each side by the LCD.
   b. Decimals: multiply all terms on each side by the largest power of 10 needed.
2) Use the Distributive Property to remove parentheses.
3) Simplify each side of the equation, by combining like terms.
4) Separate variable & constant terms – use the Addition Property
   a. Move variable terms to one side (to left side or to larger positive coefficient)
   b. Move constant terms to the other side (to right side or away from variable)
5) Move the coefficient of the variable using the Multiplication Property.
6) Check solution by substituting back into the original equation.

"What is Move?":

- Move in Step 4 above means to add the "opposite" of the term to each side of the equation. For example if we have 3 + x = 4, to move the 3, we add "- 3" to each side of the equation.
- Move in Step 5 above means we multiply both sides of the equation by the inverse of the coefficient. For example, 7x = 42, we multiply both sides of the equation by $\frac{1}{7}$.

"Why Move?" We move terms and coefficients because want to find out what x is. When we move the constant term to one side, we get the variable term by itself. When we move the coefficient of the variable, we get the variable by itself and are 100% certain of the value of the variable.

Another answer to the "Why Move" Questions is we want to have no x's on one side of the equation and all the x's on the other side. For example, with 3x + 3 = 4x want 0x + 3 = ??x then want 1x ie, ?? = x

Example 1:

$$\begin{array}{rcl} 7 + 6x & = & 2x \\ -\ 6x & & -6x \\ \hline 7 & = & -4x \end{array}$$

Subtract 6x from each side of the equation.

$$\frac{7}{-4} = \frac{-4x}{-4}$$

$$-\frac{7}{4} = x$$

Divide each side by – 4, or multiply by $-\frac{1}{4}$.

Example 2:

$$3(2x + 5) = 4x - 7$$

Distribute 3 to each term in parentheses.

$$\begin{array}{rcl} 6x + 15 & = & 4x - 7 \\ -4x - 15 & & -\ 4x - 15 \\ \hline 2x & = & -\ 22 \end{array}$$

Subtract 4x & subtract 15 from each side of the equation.

$$\frac{2x}{2} = \frac{-22}{2}$$

$$x = -11$$

Divide both sides by 2.

Example 3: 9x + 3(x – 4) = 10(x – 5) + 7

9x + 3x – 12 = 10x – 50 + 7

Step 2 (above): Distribute to remove parentheses.

12x – 12 = 10x – 43

Step 3: Combine like terms.

$$\begin{array}{rcl} 12x - 12 & = & 10x - 43 \\ -\ 10x + 12 & & -\ 10x + 12 \\ \hline 2x & = & -\ 31 \end{array}$$

Step 4: subtract 10x from each side AND add 12 to each side.

$$\frac{2x}{2} = -\frac{31}{2}$$

Step 5: Divide both sides by 2.

$$x = -\frac{31}{2}$$

Reduce fractions as necessary.

Example 4: $\frac{2x-1}{4}+\frac{x+2}{3}=3$

$12\left(\frac{2x-1}{4}+\frac{x+2}{3}\right)=12(3)$

Step 1: LCD = 12

Multiply both sides of the equation by the LCD to clear fractions.

$12\left(\frac{2x-1}{4}\right)+12\left(\frac{x+2}{3}\right)=12(3)$

Step 2: Make sure to distribute the 12 to each term on both sides of the equation.

$3(2x-1)+4(x+2)=12(3)$

Cancel/Reduce as appropriate.

6x – 3 + 4x + 8 = 36

Step 2 (again): Distribute to remove parentheses.

10x + 5 = 36

Step 3: Combine like terms.

$$\begin{array}{rcrcr} 10x & + & 5 & = & 36 \\ & - & 5 & & -\ 5 \\ \hline 10x & & & = & 31 \end{array}$$

Step 4: Subtract 5 from each side to get the variable term by itself.

$\frac{10x}{10}=\frac{31}{10}$

Step 5: Divide both sides by 10 to get variable by itself.

$x=\frac{31}{10}$

Reduce fractions as necessary.

In order to solve other types of equations, we will be using the steps to solve Linear Equations, since we will be able to isolate linear parts of our equations. We used several of the rules from earlier in this chapter on Example 4. Some problems will require us to use even more.