Compound Linear Inequalities

Notations

Inequality Notation	Interval Notation	Meaning in Words
$x < b$	$(-\infty, b)$	All numbers less than b, but not including b.
$x \le b$	$(-\infty, b]$	All numbers less than b, including b.
$a < x$	$(a, +\infty)$	All numbers greater than a, but not including a.
$a \le x$	$[a, +\infty)$	All numbers greater than a, including a.
$a < x < b$	(a, b)	All numbers between a & b, but not including a or b.
$a \le x < b$	$[a, b)$	All numbers between a & b, including a.
$a < x \le b$	$(a, b]$	All numbers between a & b, including b.
$a \le x \le b$	$[a, b]$	All numbers between a & b, including a & b.
$x < a$ & $b < x$	$(-\infty, a) \cup (b, +\infty)$	All numbers less than a AND All numbers greater than b, but not including a & b.
$x \le a$ & $b \le x$	$(-\infty, a] \cup [b, +\infty)$	All numbers less than a AND All numbers greater than b, including a & b.

Be very careful on the last 2. You may come across something like: $x < 3$ & $3 < x$. This does NOT mean $x < 3 < x$! Many people get this wrong. In this case we have $x \neq 3$. You may also come across $x < 5$, $10 < x$, again do not write $10 < x < 5$, because that would mean $10 < 5$, which we all know is not true.

Example: $-6 \le 3k - 9 \le 0$

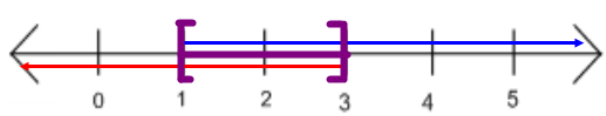
I recommend making this problem 2 inequalities, $-6 \le 3k - 9$ and $3k - 9 \le 0$. This cuts down on the possibility for the problems mentioned above. Many errors occur when students forget that each part of the inequality must use the addition or multiplication property.

$$-6 \le 3k - 9 \le 0$$
$$\underline{+9 \qquad\quad +9 \quad +9}$$
$$+3 \le 3k \le 9$$
$$\frac{+3}{3} \le \frac{3k}{3} \le \frac{9}{3}$$
$$1 \le k \le 3$$

$-6 \le 3k - 9$ & $3k - 9 \le 0$

$3 \le 3k$ & $3k \le 9$

$1 \le k$ & $k \le 3$



Example: -1 < -2x + 4 < 5

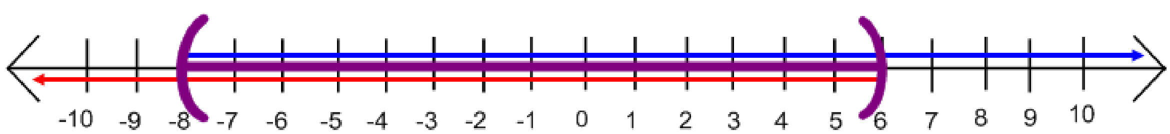
$$-1 < -2x + 4 < 5$$
$$\underline{-4 \qquad\qquad -4 \quad -4}$$
$$-5 < -2x < 1$$
$$\frac{-5}{-2} > x > \frac{1}{-2}$$
$$\frac{5}{2} > x > -\frac{1}{2} \text{ or } -\frac{1}{2} < x < \frac{5}{2}$$

Now divide by -2
AND turn the symbols!

This type of problem can be done "all at once", you just need to pay attention to every step you take!

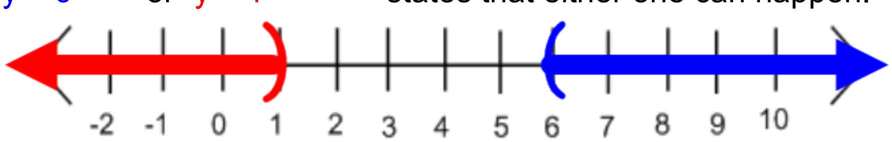
Graph:

Example: a + 6 > -2 and 5a < 30
Solve each inequality then graph the intersection, since the
a > -8 and a < 6 problem states that they both must occur.
So the solution is the interval (-8, 6)



Example: 5y > 30 or y – 3 < -2

In this case we graph the union of the intervals, since the problem
y > 6 or y < 1 states that either one can happen.



Example:
To get a "B" in a course, a student needs an average of at least an 80% but less than a 90% on 6 tests. Rowena received an 82, 76, 83, 92 and 67 (each out of 100) on the first five tests. What does she need on the sixth test to get a "B" in the course?

What we know:

This is an average problem, the average formula for this problem: $\frac{\text{sum of numbers}}{6}$

Let x be the unknown test grade.
She needs at least an 80 → 80 ≤ average.
She needs less than a 90 → average ≤ 90.

The "average" can be replaced by the formula in line 1, and the last two combined:

$$80 \le \frac{82+76+83+92+67+x}{6} \le 90$$

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The first step for solving this is to simplify the middle part.

$$80 \le \frac{400+x}{6} \le 90$$

Now we can multiply each part by 6, to remove the fraction.
Multiply.

$6*80 \le 400 + x \le 6*90$

Finally subtract 400 from each side.

$$\begin{array}{rcccl} 480 \le & & 400 + x & \le & 540 \\ -400 & & -400 & & -400 \\ \hline 80 \le & & x & \le & 140 \end{array}$$

Since the test is out of 100 she will not get more than 100.

$80 \le x \le 100$

Interval Notation: [80, 100]