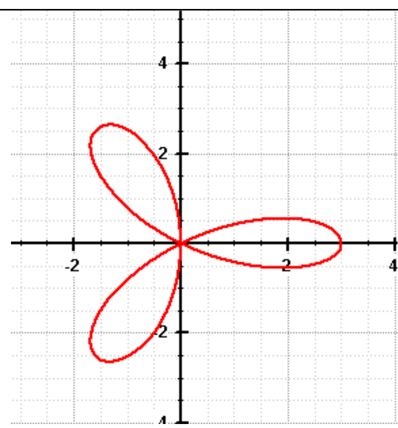
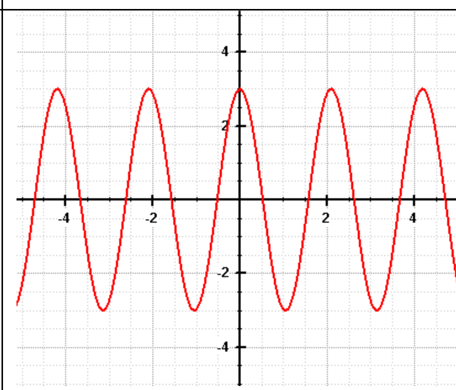
Functions

A **function** is a relation where the x-value is different for every ordered pair, but the y-value may be the same. The relation {(1, 2), (1, 1), (2, 2), (3, 4), (5, 7)} is not a function because two of the ordered pairs have 1 as the x-value. However, the relation, {(1, 2), (2, 2), (3, 4), (5, 7)}, is a function, since each x-value is different, even though two of the ordered pairs have 2 as a y-value.

On a graph you can check if a relation is a function using the **vertical line test**, a vertical line drawn through the graph will cross the graph no more than one time if the graph is of a function. In the graph examples below, a vertical line will cross the first graph twice in at least one place, thus the first graph is a relation, not a function. On the second graph any vertical line will cross the graph no more than one time, and so it is the graph of a function.

<table>
<tr><th></th><th>Relations</th><th>Functions</th></tr>
<tr><td>Equations</td><td>$x^2 + y^2 = 12$ (equation of a circle)</td><td>$x + y = 12$ (equation of a line)

or $f(x) = -x + 12$</td></tr>
<tr><td>Graphs</td><td>
</td><td>
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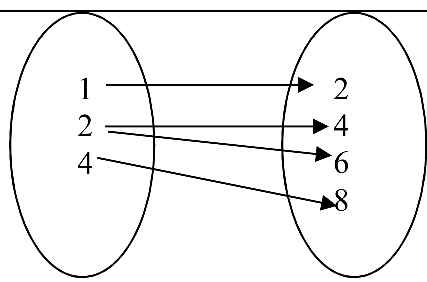
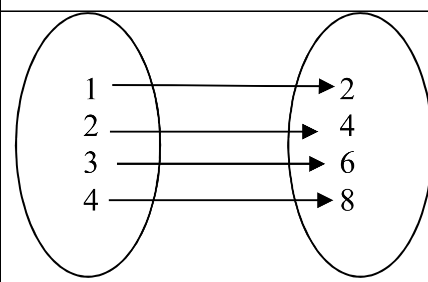
<tr><td>Diagrams</td><td>
</td><td>
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Relations and functions describe how pairs of data are related. If the relation, equation or graph is a function then we know exactly which y-value we will have depending upon the given x-value. For example, when the function describes the height of an object, from the ground, at a specific time after being thrown, we can calculate the exact height it will have at 3 seconds after being thrown.

When we speak of the function of an object, or person, that object or person has a specific task or tasks that are to be performed; this is true for a mathematical function as well. For example, the function $f(x) = x + 3$, the task that this function performs is to add 3 to x, no matter the actual value of x. We will only get one answer out for each different x value.

Functions not only describe how numbers are related, but can be used for other items as well, such as a person and their birthday, US state and its capital{(PA, Harrisburg), (CA, Sacramento), …}. An example of a constant function would be US state and country, {(PA, USA), (NJ, USA), …}

The **domain** of a function is all the x-values of the ordered pairs. We can also say that the domain is the set of x values such that the function makes sense. For example, if x is the denominator x cannot be 0.

The **range** is all the y-values of the ordered pairs. We can also say that the range is the set of y values that result from the domain values.

We sometimes call a function a map. We use it to get from the domain (starting point) to the range (ending point). Each ending point depends upon where we start. If I were to tell you to turn left, go straight 2 miles then turn right, where would you be? It would depend where you started!

> Start at the domain, follow the function to get to the range.
> Start at home, follow the map to get to your destination.

Functions may also be written as equations, or illustrated with graphs and diagrams of the correspondence. When a function is written as an equation, we often use f(x) instead of y, to show that the relation is a function of x. When a function is written as an equation or shown as a graph, the domain and range are normally intervals written in terms of x and y respectively.

NOTE: f(x) means the function evaluated at x, not f times x.

A linear function when graphed forms a line.

The linear function has a domain of $(-\infty, \infty)$ and a range of $(-\infty, \infty)$

For the basic parabola (quadratic) the domain is $(-\infty, \infty)$ and its range is $(0, \infty)$

The **independent variable** of a function is the x variable or first coordinate in the ordered pairs, the **dependent variable** is the y variable (its value depends on the x-value).

Example: Let a = length, b = perimeter, width = 5 inches. Then b depends upon a, b is a function of a, $P = 2l + 2w$, $b = 2(a + 5)$

When an equation is used to describe a function we can find the domain by ruling out certain values of x:

- Any value of x that will cause the denominator to be zero.
- Any value of x that will cause a negative to be under a square root.

When determining the range of a function we look at all the possible outputs based on the domain.

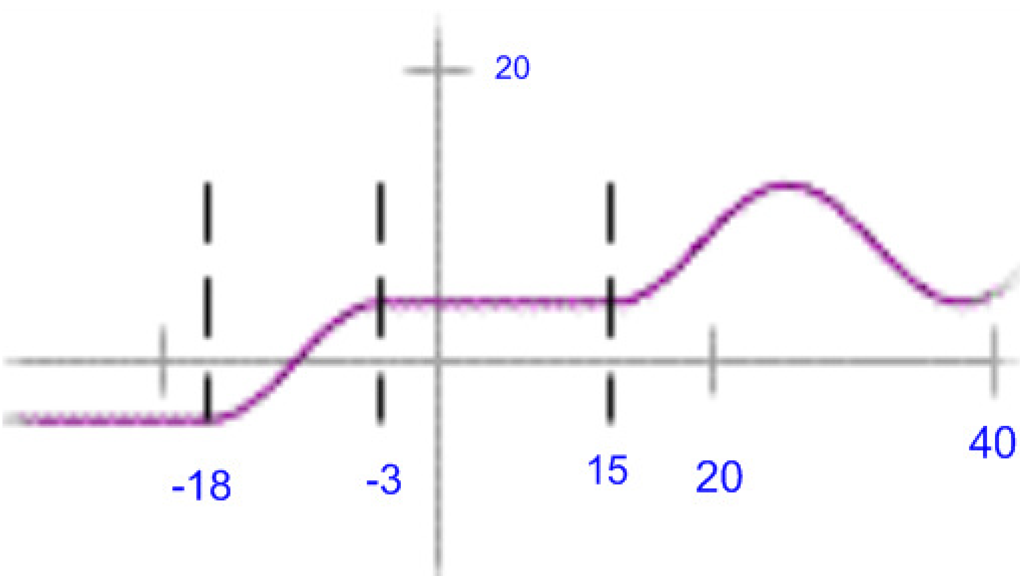
A function is **increasing** when the slope is positive, so the graph is moving up to the right, f(a) < f(b) when a < b. In other words, as you read the graph from left to right it has an upward trend.

A function is **decreasing** when the slope is negative, so the graph is moving down to the right, f(a) > f(b) when a < b. In other words, as you read the graph from left to right it has a downward trend.

A function is **constant** when the slope is zero, so the graph is horizontal, f(a) = f(b) when a < b. In other words, as you read the graph from left to right it is horizontal. Only the constant function below has an interval over which it is constant, since the interval must be an open interval.

NOTE: a < b is the open interval (a, b)

Example:



The function above is constant on the intervals: $(-\infty,-18)$ & $(-3,15)$.
The function above is increasing on the intervals: $(-18,-3),(15,22),(39,\infty)$
The function above is decreasing on the interval: (22, 39)

NOTE: If we union the intervals above, we will have the domain of the function! This is always true.

Non-linear functions can have more than 1 of the above types of slope over different intervals. Linear functions can only be increasing, decreasing OR constant.

Most functions are in at least one of the above states. Many functions have more than one state which depends upon the interval. The interval when stated, refers to the x-value of the function and is normally written in interval notation, though sometimes set builder notation is used. The chart of some basic functions, in the file “Graphs of Functions”, includes the intervals on which they are increasing, decreasing or constant.

The **x-intercept(s)** of a function are the real solutions to the equation f(x) = 0. When the function is already graphed, the x-intercepts are where the function crosses the x-axis. If you are working with an equation, it is necessary to replace y, f(x) with 0 and solve for x. Details for solving these equations will be available as they are covered throughout the course. We covered x-intercepts of linear equations in Chapter 2

The **y-intercept** of a function is when x = 0, or when the function is evaluated at 0, i.e. f(0). When the function is graphed the y-intercept is where the function crosses the y-axis. If you are working with an equation, just evaluate the equation at x = 0, and you will have your y-intercept. We covered y-intercepts of linear equations in Chapter 2. A function will never have more than one y-intercept, if it had more than one:

- it would fail the vertical line test, using the y-axis as a vertical line.
- each value of x can only give one value of y.

NOTE: Not all lines are functions. x = 3 is a vertical line, which will not pass the vertical line test and so is not a function.

The **average rate of change** for a function, is the slope between any two points on the function, $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$ remember f(x) = y…, so this is the slope formula we examined with linear equations.

Graphs were made using WZGrapher from www.walterzorn.com